Predation and Sectorial Composition along Development*

Carlos Betherncourt[†]
Universidad de la Laguna

Fernando Perera Tallo
Universidad de la Laguna

October 10, 2014

Abstract

Predation attracts a relatively high portion of labor in developing countries and obstructs development. We formulate a model in which agents devote time either to predation or to producing agricultural and manufactured goods with the following features: a subsistence level of agricultural goods must be reached and, consequently, poor countries devote more resources to agriculture; agriculture is more land intensive and, thus, has a lower labor share than manufacturing; and incentives to devote time to production increase with the labor share. A structural change occurs throughout the transition: the share of manufactured goods in GDP increases, raising the labor share and discouraging predation. This mechanism involves an amplification effect of the differences in productivity among countries due to the reallocation of labor from predation to production. Finally, institutional quality plays a crucial role in this process, since it discourages predation and fosters the labor reallocation.

*We thank the participants at the XVI conference on Dynamics, Economic Growth, and International Trade (DEGIT-XVI) held in the SPbU Saint Petersburg for their helpful comments. We specially thank José-Víctor Ríos-Rull, Costas Azariadis, David Mayer-Foulkes and Jakub Growiec. Both authors are members of CAERP and they wish to acknowledge the support and stimulating environment of this centre.

[†]Corresponding author. Universidad de La Laguna, Campus de Guajara s/n, La Laguna, Tenerife, Spain. Tel.: +34 922317954; fax: +34 922 317204. E-mail address: cbethenc@ull.es.

1. Introduction

Many developing countries fail to achieve a successful development process (see Quah 1996, 1997 and Parente and Prescott, 1993). Much effort in current macroeconomic research has been devoted to explaining this fact and new features such as the nature and composition of the economic activities have been explored. In this regard, it is well known that in economies, resources are devoted to both productive activities (resources are used in the production of goods and services) and unproductive activities (resources are used to generate rents, i.e., income but not goods). Thus, unproductive activities entail a group of activities that share the common feature of being profitable, but wasteful, for example, property crime, fraud, begging, lobbying, rent-seeking, etc. We will call all these unproductive activities predation from now on. More precisely, we consider that predation is any activity in which an agent, acting as a predator, uses factors to capture the production generated from others, the preys.

Empirical evidence suggests that the size of the unproductive sector is larger in low income countries. Although many predation activities are legal, the ones that are better measured are the illegal ones. For example, the share of the criminal predatory sector in GDP is 20.7% for Latin America, while it is 6.89% for the United States[1]. Another example in the literature is from Bourguignon (1999), who finds that the share of property crime in GDP is 1.5% for Latin America, while it is 0.5% for United States. In a recent contribution Soares and Naritomi (2010) show, for a wide sample of countries, that regions with higher GDP per capita, such as North America and Western Europe, also display lower burglary and theft rates. Considering that corruption is defined as the abuse of public office for private gain then, a broad range of actions as bribery or embezzlement can be identified as pure acts of predation. In this respect, Treisman (2000), Paldam (2001, 2002), Brunetti and Weder (2003) and Rehman and Naveed (2007), among others, evidence that corruption is higher in less developed and developing countries. Moreover, countries which show high levels of corruption, usually display high levels of other forms of predation. For instant, Morck et al. (2000) find that more corrupt countries display also more price manipulation. Finally, the existence of measurement problems explains why there is not much research on quantitative analysis of predation derived from legal activities. In this respect, Murphy, Shleifer, and Vishny (1991) even suggests the possibility of consider-

[1] These numbers are calculated using the studies of Anderson (1999) and Londoño and Guerrero (1998).

ing the proportion of students concentrating in law as a proxy of the size of the predatory sector.

Similarly, many studies document significant differences on the factorial allocation among developed and developing countries. If we look at the sectorial composition of countries, empirical evidence seems to suggest that while developed countries use more hi-tech machinery in the production of agriculture, the portion of land devoted to agriculture is much larger in developing countries. For example, the last report of the World Bank (2014) finds that the number of tractors per 100 squared km of arable land are 373.1 and 96.9 respectively for both high income and low and middle income countries whereas the percentage of land used in agriculture is 43% for low and middle income and 30% for high income countries. Nevertheless, differences of the factorial allocation between developed and developing countries result stronger when we look at the allocation of factor labor. The same report of the World Bank documents that the total employment in agriculture is about 48.5% in low and middle income and only 5.4% in high income countries. These numbers imply that the percentage of workers in agriculture in developing countries almost 10 times the percentage of the developed ones.

At the same time, many recent studies have found agriculture to be less labor intensive than both industry and services while the capital intensity is similar for all sectors. Echevarria (1998) finds that in Canada the labor share represents 41% of value added in agriculture, 59% of value added in industry and 51% of value added in services; whereas the capital share represent 43% of value added in agriculture, 41% of value added in industry and 49% of value added in services. More recently, Valentinyi and Herrendorf (2008) finds that while the agriculture shows the highest land share in US, around 11% of the value added in agriculture and less that 0.5% in the remaining sectors; the capital share is similar among sectors being 31% of value added in agriculture, 33% in manufactured consumption and 35% in services.

This paper proposes a mechanism that connects the empirical facts mentioned above which is formalized throughout a three sector neoclassical growth model. Workers devote time to producing agricultural and manufactured goods and to predation. As agricultural goods satisfy primary necessities, a "food problem" arises: a subsistence level of agricultural goods is required before the consumption of manufactured goods takes place. Thus, low income countries devote a higher portion of their resources to agriculture than high income countries. Moreover, since the capital share is similar among sectors and the agricultural sector is more land intensive than the manufacturing one, this implies that the agricultural sector shows a lower labor share than the manufacturing sector. These two features together, the higher weight of agriculture in the production of low income countries and the lower labor share in agriculture imply that the aggregate labor share is lower in poor countries than in rich countries. Low labor shares imply low rewards for work relative to predation, discouraging work in productive activities and stimulating predation. Thus, low income countries are characterized by a high weight of agriculture in production, a low labor share and a high level of predation. As the country accumulates capital and subsistence needs lose weight in households' budgets, a structural change occurs in which more resources are devoted to produce manufactured goods. This structural change implies a rise in the weight of the manufacturing sector, which gives rise to an increasing aggregate labor share during the transition to the steady state. This increase in labor share raises the relative reward of working in productive activities, thus encouraging the reallocation of labor from predation to production. Summarizing, a structural change occurs throughout the transition from an initial per capita capital lower than the steady state level during which predation falls and the weight of agriculture declines in favor of manufacturing.

This paper shows that differences in institutions are not a unique explanation for differences in levels of predation among countries. The reallocation of resources generated by the change in sectorial composition during the transition to the steady state plays an important role accounting for differences in predation. At the initial stage of development the portion of labor employed in agriculture is high, implying a low level of labor share. However, insofar countries accumulate capital during the transition and the "food problem" is being solved, labor and capital start to be reallocated in manufacture. The larger size of the manufacture owing a higher labor share than the agriculture, implies that the aggregate labor increases. Thus, the incresing weight of the manufacture (and the decreasing weight of the agriculture) implies an increase of the aggregate labor share during the transition to the steady state when the initial per capita capital is lower than the steady state level. Since a lower labor share results in fewer incentives to devote time to production and more incentives for predation, agents therefore devote more time to predation when per capita income is low, and consequently predation declines during the transition to the steady state when the initial per capita capital is lower than the steady state level. Thus the paper shows that a feedback mechanism emerges: predation affects capital accumulation, reducing the return on savings and damping capital accumulation and, capital accumulation affects predation by expanding the manufacturing sector and the labor share, which, in turn, discourages predation. predation discourages capital accumulation, and capital accumulation discourages predation. This new approach contrasts with the standard literature, which traditionally has presented differences in institutions as the sole explanation, and considers that institutions may affect factor accumulation but not the other way around. In this respect, recent empirical studies such as Glaeser et al. (2004) and Djankov et al. (2003) support our hypothesis that predation is affected by not only institutions but also factor accumulation.

This paper also contributes to explain why differences in per capita income among countries have remained stable. It is widely accepted that differences in TFP are one of the main sources of differences in per capita income[2]. The paper proposes a mechanism that involves the reallocation of resources from predation to productive activities and the incentives to engage in these activities; and that amplifies differences in TFP and per capita income generated by technological differences across countries.This mechanism is in line with the empirical research that emphasizes the differences in "social infrastructure", using the terminology by Hall and Jones (1999), to understand differences in TFP across countries, instead of the more conventional view, which considers these differences as mere technological ones. The mechanism works as follows: when productivity (in manufacturing or agriculture) rises, there is a positive direct effect on production and an indirect effect due to the accumulation of capital (the rise in productivity increases the return on savings and so, the incentives to accumulate more capital). However, together with these standard mechanisms, in the current model there exists another additional mechanism which amplifies the effect of productivity on per capita income. This new mechanism is related to predation and the change in the sectorial composition: when productivity rises, the per capita capital rises and resources are reallocated from agriculture to manufacturing. The larger the relative size of manufacturing, the greater the aggregate labor share, reducing the incentive to predate and increasing the portion of labor devoted to production (agriculture and manufacturing). This increase in the amount of labor devoted to production has three positive effects on the per capita income: *i*) a direct effect on per capita production; *ii*) an indirect effect due to the accumulation of capital: when labor rises, it increases both the marginal productivity of capital and the incentive to accumulate more capital; and *iii*) a reduction in the portion of labor devoted to predation which implies an increase in the share of the marginal product of capital that goes to savers, raising the return on savings and promoting the

[2]See Easterly and Levine (2001), Hall and Jones (1999), and Parente and Prescott (2000).

accumulation of capital.

Finally, according to the literature which emphasizes the role of differences in institutions to explain differences in per capita income[3], we study the effect of an improvement in the quality of the institutions to deter predation and the consequences for development. We show that institutional quality is a crucial factor for development. In particular, we find that an improvement in the institutional quality reduces the productivity of predation, generating a labor reallocation from predation to productive activities which produces the same three positive effects on per capita income, as described above.

This paper is organized as follows. Section 2 summarizes the relevant literature of main facts. Section 3 develops a model of three sectors: agriculture, manufacturing and predation. Section 4 analyzes agents' decisions and section 5 defines the equilibrium. Section 6 explains how the labor share and predation evolve with the per capita capital level. Section 7 presents the dynamic behavior of the economy. Section 8 analyzes how predation amplifies differences in productivity across countries and section 9 analyzes the role of institutions. The last section, section 10, concludes and appendix presents proofs and technical details.

2. Related literature

There exists a considerable number of papers on structural transformation which analyze the process of industrialization and the sectorial composition change (see for example Restuccia, Yang and Zhu, 2008, Gollin, Parente and Rogerson, 2002, 2004, 2007, and Córdoba and Ripoll, 2009), however, no one of them deal with predation and with the allocation of factors among different activities, including improductive activities.

On the other hand, there is a large amount of literature focused on studying the allocation of resources among productive and unproductive activities (see for example, Murphy, Shleifer, and Vishny, 1991, 1993, Acemoglu, 1995, Acemoglu and Verdier, 1998, Schrag and Scotchmer, 1993, Grossman and Kim, 1996, 2002, and Chassang and Padró-i-Miquel , 2010). Moreover, there exists another strand of the literature that analyse the relationship between social conflict and development. In this respect, Chassang and Padró-i-Miquel (2009) study the opportunity cost for predation in a repeated game. Tornell and Lane (1999) show that under weak institutions, the interaction between powerful groups may cause redistribu-

[3] See Acemoglu et al. (2005) for a complete survey.

tive distortionary fiscal policies consisting in draining resources from an efficient sector to an inefficient one. Nevertheless, no one of them analyze consequences for growth derived from the allocation of resources among productive and unproductive activities.

The papers most related to ours are the ones that establish a connection between predation and the factorial composition of income. Dal Bó and Dal Bó (2011), using a static general equilibrium setting, show that if predation is labor intensive relative to the whole economy, favorable shocks in the labor-intensive productive sector reduce predation. However, like previous contributions, their results are derived from a static framework without capital accumulation and thus, they also cannot account for the feedback process described above. The link between labor share and predation has been analyzed already in previous contributions by Zuleta (2004) and Andonova and Zuleta (2009). However, labor share in these papers is constant and, consequently, there is no feedback process between capital accumulation and predation. Furthermore, this feedback process takes place throughout the transition to the steady state, which is the focus of our paper, and this is not analyzed in previous contributions, which are centered on the steady state (Zuleta, 2004) or propose static models (Andonova and Zuleta, 2009). An excemption is Bethencourt and Perera-Tallo (2014). They propose a dynamic model where the elasticity of substitution between labour and capital is less than 1. In this setting, insofar the economy accumulates capital, the labor share increases, discouraging predation and fostering even more capital accumulation.

The relationship between labor share and development is a key factor to account for the the feedback mechanism described above. This relationship has received a lot of attention recently. National accounts' statistics typically reveal that labor share is smaller in low income countries. However, Gollin (2002) pointed out that national account labor shares are underestimated, since self-employed incomes are computed as capital income. This problem is particularly severe for developing countries, where the portion of self-employed in the labor force is quite high. Gollin proposed a set of adjustments to conventional calculations which consist of including some part of self-employment income in labor share. Gollin's preferred adjustment is based on the assumption that the labor income of self-employed is equal to the average wage of employees. The literature on self-employment in developing countries shows that, typically, self-employed workers in developing countries are poor, with low levels of education and with most of them working in the informal sector in small scale businesses, which require low skills (Banerjee and Duflo, 2007; Mel et al., 2008; Temkin, 2009; Narita, 2011). Thus, the adjustment proposed by Gollin has an upward bias for developing countries since the shadow wage of the self-employed is below that of the employees' wage (as Maarek, 2010, pointed out and the above empirical literature confirms). In addition, since most self-employed in developing countries are engaged in the informal economy, it is likely that part of their production is not accounted for in the GDP (the denominator of the labor share). In spite of this upward bias in the labor share of developing countries, it still remains lower than in developed countries after the adjustment proposed by Gollin. More precisely, the average labor share of developing countries reported by Gollin is 0.584, while the average in developed countries is 0.687[4]. One obvious limitation in Gollin's analysis is the small data set used in which developing countries are under-represented. Harrison (2005) using a similar methodology, but with a much larger data set with respect to both the number of countries and number of years, confirms that there are significant differences in labor share between developing and developed countries, with developing countries displaying lower shares. Another approach is to use industrial data. This approach has the advantage that the weight of the self-employment in the sample is negligible or insignificant. Results show that using such an approach, there exists a clear and positive relationship between labor share and development indicators, such as per capita income (Ortega and Rodriguez, 2006) or capital accumulation (Decreuse and Maarek, 2009; and Maarek, 2010). Thus, all the three empirical methodologies: conventional national account calculation; adjusted national account calculation to incorporate self-employed; and the industrial data approach confirm that developing countries exhibit lower labor shares than developed ones.

3. The model

Time is continuous with an infinite horizon. There are two different goods in the economy: agricultural and manufactured goods, denoted by subindexes a and m respectively. Agricultural goods are used only for consumption, while manufac-

[4] To calculate these averages, we consider developing countries to be those in which the per capita GDP reported by Gollin was smaller than US $ 6,000 (1985 as basis year) and developed countries as those above this threshold (GDPs reported by Gollin were mostly from 1992 with 1985 being the basis year). This classification coincides with the one used by the IMF and the World Bank (the World Bank uses the terminology low and middle income countries for developing countries and high income countries for developed ones).

tured goods are used for consumption and investment in physical capital:

$$Y_a(t) = C_a(t) \tag{1}$$
$$Y_m(t) = C_m(t) + \dot{K}(t) + \delta K(t) \tag{2}$$

where $Y_a(t)$ denotes the aggregate production in agriculture, $C_m(t)$ denotes the aggregate consumption in agriculture, $Y_m(t)$ denotes the aggregate production in manufacturing, $C_m(t)$ denotes the aggregate consumption in manufacturing, $K(t)$ denotes aggregate capital and $\delta \in (0,1)$ denotes depreciation rate. $\dot{K}(t) + \delta K(t)$ denotes the gross investment.

3.1. Technology

Production technologies of agricultural and manufactured goods are given by the following production functions:

$$Y_a(t) = \Gamma_a (K_a(t))^{\alpha} (Z_a(t))^{\beta} (L_a(t))^{1-\alpha-\beta} \tag{3}$$
$$Y_m(t) = \Gamma_m (K_m(t))^{\alpha} (L_m(t))^{1-\alpha} \tag{4}$$

where $K_a(t)$ and $K_m(t)$ denote, respectively, the physical capital used in agriculture and manufacturing; $L_a(t)$ and $L_m(t)$ the amount of labor used in agriculture and manufacturing, and $Z_a(t)$ denotes the amount of land used in agriculture. In order to capture the fact that agriculture is more land intensive than manufacturing, we have used the extreme but simple assumption that only agriculture uses land. Technologies also reproduce the empirical fact that the labor share of the agricultural sector is smaller than the labor share of manufacturing.

The per capita production of the agricultural and manufacturing sectors are given by:

$$y_a = \Gamma_a k_a^{\alpha} {z_a}^{\beta} {l_a}^{1-\alpha-\beta} \tag{5}$$
$$y_m = \Gamma_m k_m^{\alpha} {l_m}^{1-\alpha} \tag{6}$$

3.2. Preferences

The economy is populated with many identical dynasties of homogeneous agents. To simplify, we assume that population is constant. Preferences of a dynasty are given by the following function:

$$\int_{\tau}^{\infty} \ln\left(c(t) - \overline{c}\right) e^{-\rho(t-\tau)} dt, \quad c(t) = \begin{cases} c_a(t) & \text{if } c_a(t) \leq \overline{c} \\ \overline{c} + c_m(t) & \text{if } c_a(t) \geq \overline{c} \end{cases}$$

where $c_a(t)$ and $c_m(t)$ denote, respectively, the per capita consumption of dynasty of agricultural and manufactured goods in period t, and $\rho > 0$ is the discount rate of the utility function. Thus, these preferences imply a "food problem": households do not consume manufactured goods until reaching a certain "subsistence" level of consumption of agricultural goods, denoted by $\overline{c}$.

3.3. The predation technology

Each period, agents are endowed with fixed z units of land and one unit of time which can be devoted to undertake two types of economic activities: to produce goods (agricultural or manufactured goods), l, and to commit predation, l_p, that is,

$$1 = l(t) + l_p(t) \tag{7}$$

We define predation activity as any activity which implies the use of resources to obtain incomes without generating production. We include property crimes, fraud, corruption, lobbying, etc. The amount of income obtained through predation is denoted by $\widetilde{y}(t)g\left(l_p\right)$, where $\widetilde{y}(t)$ is the per capita production and $g : \Re_+ \rightarrow [0,1]$ is the fraction of per capita gross production that each agent predates, which depends positively on the amount of time devoted to such activity, l_p. We assume that the function $g(.)$ is strictly increasing, strictly concave, continuous and differentiable of the second order and $g(0) = 0$, $g(1) < 1$ and $g'(0) \geq 1$.

4. Agents' decisions

We will concentrate on the case in which the economy has solved the "food problem", this is, when the consumption is above subsistence level and therefore, the consumption of manufactured good is positive.

4.1. Households:

Household maximization problem is as follows:

$$\max_{\{c(t),l(t),l_p(t),b(t)\}_{t=0}^{\infty}} \int_0^{\infty} \ln(c(t)-\overline{c})e^{-\rho t}dt \tag{8}$$

$$\begin{aligned}
&s.t: \\
c(t) &= c_m(t)+\overline{c} \qquad (9)\\
\dot{b}(t) &= \underbrace{w(t)l(t)+r(t)b(t)+w_z(t)z\text{-}g(\widetilde{l}_p(t))y(t)}_{\text{Net income from production}}+\underbrace{g(l_p(t))\widetilde{y}(t)}_{\text{Predation income}}\text{-}c_m(t)\text{-}p_a(t)\overline{c} \\
l(t)+l_p(t) &= 1 \\
y(t) &= w(t)l(t)+(\delta+r(t))b(t)+w_z(t)z
\end{aligned}$$

where $b(t)$ denotes the amount of assets of the household, $w(t)$ the wage per unit of labor, $r(t)$ the *net* return on assets, $w_z(t)$ the renting price of land, $y(t)$ the household's *gross* income and $p_a(t)$ the price of agricultural goods in terms of manufactured goods. We normalize the price of manufactured goods to one. Since $r(t)$ is the net return on assets, $\delta+r(t)$ is the gross interest rate, which is the one that appears in the definition of gross income. The sign "$\sim$" over a variable means that such variable is a per capita variable of the economy and therefore, the household cannot decide on it. Thus, $\widetilde{l}_p$ denotes the per capita labor devoted to predation and $\widetilde{y}$ denotes the per capita gross income. Income coming from the production sector is equal to labor income from the production sector $w(t)l(t)$, plus financial income $r(t)b(t)$, plus land rents $w_z(t)z$, minus the amount of this income that is predated by other agents in the economy $g(\widetilde{l}_p(t))y(t)$. The other source of income comes from the predation sector which is equal to $g(l_p(t))\widetilde{y}(t)$. It is straightforward from the definition of preferences that when an agent enjoys a consumption level above the subsistence consumption level, she is going to consume the subsistence level of agricultural goods $\overline{c}$. Thus, the total expenditure in consumption is equal to the expenditure in agricultural goods, $p_a(t)\overline{c}$, plus the expenditure in consumption of manufactured goods $c_m(t)$. The increase of the household's assets, $\dot{b}(t)$, is equal to its savings, which is equal to its income (the one from production plus the one from predation) minus the expenditure in consumption goods, $c_m(t)+p_a(t)\overline{c}$.

The first order conditions for the interior solution are as follows:

$$w(t)\left[1-g(\widetilde{l}_p(t))\right] = g'_{l_p}(l_p(t))\widetilde{y}(t) \tag{10}$$

$$\frac{\dot{c_m}(t)}{c_m(t)} = (r(t)+\delta)\left(1-g(\widetilde{l}_p(t))\right)-\delta-\rho \tag{11}$$

Equation (10) specifies that the net wage in the production sector after predation should be equal to the marginal payment of predation activities. That is, the marginal payment of the time devoted to each activity should be alike. Equation (11) is the typical Euler equation: the speed at which consumption grows depends positively on the return on savings, $(r(t)+\delta)\left(1-g(\widetilde{l}_p(t))\right)-\delta$ and negatively on the discount rate of the household, ρ.

The following transversality condition should also be satisfied:

$$\lim_{t\to+\infty}\frac{1}{c_m(t)}e^{-\rho t}b(t)=0$$

4.2. Firms:

Firms maximize profits. The optimization problem of firms in agriculture in per capita terms is defined by:

$$\begin{aligned} \max_{y_a,l_a,z_a,k_a} \quad & p_a y_a - wl_a - w_z z_a - (\delta+r)k_a \\ s.t.: \quad & \Gamma_a k_a^{\alpha} z_a^{\beta} l_a^{1-\alpha-\beta} \geq y_a \end{aligned} \tag{12}$$

while the optimization problem of firms in manufacturing is given by:

$$\begin{aligned} \max_{y_m,l_m,k_m} \quad & y_m - wl_m - (\delta+r)k_m \\ s.t.: \quad & \Gamma_m k_m^{\alpha} l_m^{1-\alpha} \geq y_m \end{aligned} \tag{13}$$

The first order conditions of the above problems are:

$$
\begin{aligned}
(1-\alpha-\beta)p_a\Gamma_a\frac{k_a^\alpha {z_a}^\beta {l_a}^{1-\alpha-\beta}}{l_a} &= w && (14)\\
\alpha p_a\Gamma_a\frac{k_a^\alpha {z_a}^\beta {l_a}^{1-\alpha-\beta}}{k_a} &= (\delta+r) && (15)\\
\beta p_a\Gamma_a\frac{k_a^\alpha {z_a}^\beta {l_a}^{1-\alpha-\beta}}{z_a} &= w_z && (16)\\
(1-\alpha)\Gamma_m\frac{k_m^\alpha {l_m}^{1-\alpha}}{l_m} &= w && (17)\\
\alpha\Gamma_m\frac{k_m^\alpha {l_m}^{1-\alpha}}{k_m} &= (\delta+r) && (18)
\end{aligned}
$$

These conditions are the standard conditions of optimization and indicate that firms hire a factor until reaching the point at which the marginal productivity of the factor is equal to its price.

5. Equilibrium Definition

The definition of equilibrium is standard: equilibrium occurs when agents maximize their objective functions and markets clear. Steady state equilibrium is an equilibrium in which both the allocation and prices always remain constant over time.

Definition 1. *An equilibrium is an allocation* $\{c_m(t), c_a(t), l(t), l_p(t), b(t), y_a(t), z_a(t),$ $l_a(t), k_a(t), y_m(t), l_m(t), k_m(t), \widetilde{l}_p(t), \widetilde{y}(t)\Big\}_{t=0}^{\infty}$ *and a vector of prices* $\{p_a(t), w(t), r(t), w_z(t)\}_{t=0}^{\infty}$ *such that* $\forall t$ *the following conditions hold:*

- Households maximize their utility, that is, $\{c_m(t), l(t), l_p(t), b(t)\}_{t=0}^{\infty}$ is the solution of the household's maximization problem (8) and $c_a(t) = \overline{c}$.
- Firms maximize profits, that is, $\forall t$ $y_a(t)$, $l_a(t)$, $z_a(t)$, $k_a(t)$ and $y_m(t)$, $l_m(t)$, $k_m(t)$ are the solution of the optimization problem of firms (12) and (13).
- Capital market clears: $\forall t$ $k_a(t) + k_m(t) = k(t) = b(t)$.
- Labor market clears: $\forall t$ $l_a(t) + l_m(t) = l(t)$.

- Land market clears: $\forall t \; z_a(t) = z$.
- Good Market clears: $\bar{c} = y_a(t)$, $c_m(t) + \dot{k}(t) + \delta k(t) = y_m(t)$.
- Finally, since households are identical, per capita variables coincide with household variables: $\forall t \; \widetilde{l_p}(t) = l_p(t)$ and $\widetilde{y}(t) = w(t)l(t) + (\delta + r(t))b(t) + w_z(t)z$.

Definition 2. *Steady state equilibrium is an equilibrium in which both the allocation and prices always remain constant over time.*

6. Predation and per capita capital

6.1. Labor share

We define the labor share, λ, in the productive sector as the fraction of labor income over the value of the production:

$$\lambda = \frac{wl}{y} = \frac{wl_a + wl_m}{p_a y_a + y_m}$$

and we denote the portion of productive labor devoted to agriculture by $\psi_a \equiv l_a/l$.

Lemma 3. *Labor share is a decreasing function of the portion of productive labor devoted to agriculture, ψ_a.*

Labor share in the economy is a weighted average of the labor share in the manufacturing and agricultural sectors, where the weight of each sector is equal to the portion of the value of production that each sector has in the aggregate GDP. If a higher portion of productive labor is devoted to agriculture, then a higher portion of GDP comes from the agricultural sector, and this reduces labor share.

Lemma 4. *The portion of labor devoted to predation, l_p, is a strictly decreasing function of labor share, with $l_p = 1$ when the labor share is equal to zero, $\lambda = 0$, and $l_p = l_p^{\min} \leq 1$ when the labor share is equal to one, $\lambda = 1$.*

Thus, a higher labor share increases the relative reward for work with respect to predation, which encourages work in productive activities and discourages predation.

6.2. Labor devoted to predation and per capita capital

Proposition 5. *The portion of labor devoted to agriculture at equilibrium, ψ_a, and the portion of labor devoted to predation at equilibrium, l_p are strictly decreasing functions of k, Γ_a and z, and a strictly increasing functions of $\overline{c}$. The portion of labor devoted to production at equilibrium, l, is a strictly increasing function of k, Γ_a and z, and a strictly decreasing function of $\overline{c}$.*

Households' preferences imply that households do not consume manufactured goods until reaching a certain "subsistence" level of consumption of agricultural goods, $\overline{c}$. When the resources of the economy (per capita capital or land) expand or agricultural technology improves, it increases the number of resources, including labor, to be allocated in the manufacturing sector. This increases labor share, discouraging predation and fostering work in productive activities. Exactly the opposite effects occur if the subsistence level of consumption goes down.

From now on, we will denote by $l_p(k)$ and $l(k)$ and $\psi_a(k)$ the functions that relate, respectively, to the amount of labor devoted to predation, the amount of labor devoted to production and the portion of productive labor devoted to agriculture in equilibrium with the per capita capital, k.

7. Dynamic Behavior

The dynamic system that defines the dynamic behavior of the economy is as follows:

$$\begin{aligned}
\dot{k}(t) &= y_m\left(k(t)\right) - c_m(t) - \delta k(t) \\
\frac{\dot{c_m}(t)}{c_m(t)} &= \left(r\left(k(t)\right) + \delta\right)\left(1 - g\left(l_p\left(k(t)\right)\right)\right) - \delta - \rho
\end{aligned}$$

where $y_m\left(k\right)$ is the function that relates per capita production of the manufacturing sector with per capita capital at equilibrium. This function takes into account the fact that, at equilibrium, some resources of the economy are devoted to the production of agriculture and others to predation. $r\left(k(t)\right)$ is the function that relates the interest rate at equilibrium to per capita capital. These two functions are defined in the appendix in the Dynamic System subsection.

Lemma 6. *The production of the manufacturing sector at equilibrium is a strictly increasing function of the capital: $y_m : [k^{\min}, 0] \rightarrow \Re_+$. The net interest rate*

$r\left(k(t)\right)$ is a strictly decreasing function of the capital. If $\lambda_a > \alpha(1-\beta)$, then the interest rate after predation $\left(r\left(k(t)\right)+\delta\right)\left(1-g\left(l_p\left(k(t)\right)\right)\right)$ is a strictly decreasing function of the capital.

Lemma 6 states that the net interest rate that savers receive (after predation) is a decreasing function of per capita capital.

Lemma 7. *There exists $\Omega \in \Re_{++}$ such that if $\frac{\bar{c}}{\Gamma_a z^\beta} < \Omega$ then, $k^{\max} > k^{\min} > 0$ exists and: (i) $\delta + r\left(k^{\min}\right) > \rho$ and (ii) when $k \in \left(k^{\min}, k^{\max}\right)$ then $y_m\left(k\right) > \delta k$.*

7 establishes a sufficient condition to guarantee the existence of a steady state in which the consumption of manufactured goods is positive. We assume that $\frac{\bar{c}}{\Gamma_a z^\beta} < \Omega$.

Corollary 8. *If $\lambda_a > \alpha(1-\beta)$ there is a unique steady state with a positive amount of consumption of manufactured goods.*

From lemmas 6 and 7 it is straightforward to prove that the net interest rate equalizes the discount rate of the utility just once. Therefore, there is a unique steady state with a positive amount of manufactured goods.

We will concentrate our analysis on the case in this case. Thus, we assume $\lambda_a > \alpha(1-\beta)$.

Phase diagram in Figure 1 shows that the dynamic behavior of the economy is characterized by the typical saddle point dynamic[5]: there is a unique path which converges to the steady state. This means that, given the initial level of per capita capital, there is a unique equilibrium path, which converges to the steady state. When the initial amount of per capita capital is lower than the steady state level, the consumption and the portion of labor devoted to production grow throughout the equilibrium path, converging to their steady state levels, while the amount of labor devoted to predation goes down. When the amount of per capita capital is larger than the steady state level the opposite happens. Thus, when the

[5] See appendix for technical details.
****
****
NO DECIMOS NADA DE ESTO!!!
ES NECESARIO DECIR ALGO, COMO SON LAS LINEAS DE FASE, ALGO...?
****
****

starting per capita capital is below the steady state level, a "structural change" throughout the transition arises: there is a reallocation of labor from agricultural and predation sectors to the manufacturing sector. This result is consistent with the empirical literature which finds that percentage of workers in agriculture in developing countries is much higher than the percentage in the developed ones.

8. Predation as an amplification mechanism

8.1. The effect of an improvement in the technology of the agricultural sector

Phase diagram in Figure 2 displays the dynamic effect of an improvement in the technology of the agricultural sector, Γ_a. When there is an improvement in the technology of this sector, Γ_a, the amount of resources required to produce the subsistence level of consumption reduces. As a consequence, part of the capital and labor devoted to producing the subsistence level of consumption in the agricultural sector are not flowing to the manufacturing sector, increasing the amount of resources available there. However, the amount of new resources reallocated in manufacturing increases even more. An *amplification effect* of the factorial allocation emerges due to predation: when the amount of resources devoted to the agricultural sector falls, the labor share of the economy rises and this encourages the use of labor for productive activities versus predation, increasing further the production of the manufacturing sector. Moreover, this amplification effect involves a rise in the return on savings, due to the fact that the fall in predation has a direct positive effect on the portion of the return on capital that goes to savers, encouraging capital accumulation and moving the $\dot{c}$ locus to the right. The increase in the production of the manufacturing sector is reflected in the movement of the $\dot{k}$ locus, which goes up. As a consequence, the economy moves towards the new steady state with a higher level of capital, a lower portion of labor devoted to predation and a higher portion of labor and capital devoted to manufacturing. Throughout the transition a "structural change" emerges: there is a flow of labor from agricultural and predation sectors to the manufacturing sector.

8.2. The effect of an improvement in the technology of the manufacturing sector

The effect of an improvement in the technology in the manufacturing sector, Γ_m, is similar to the improvement in the technology of the agricultural sector. In fact, we

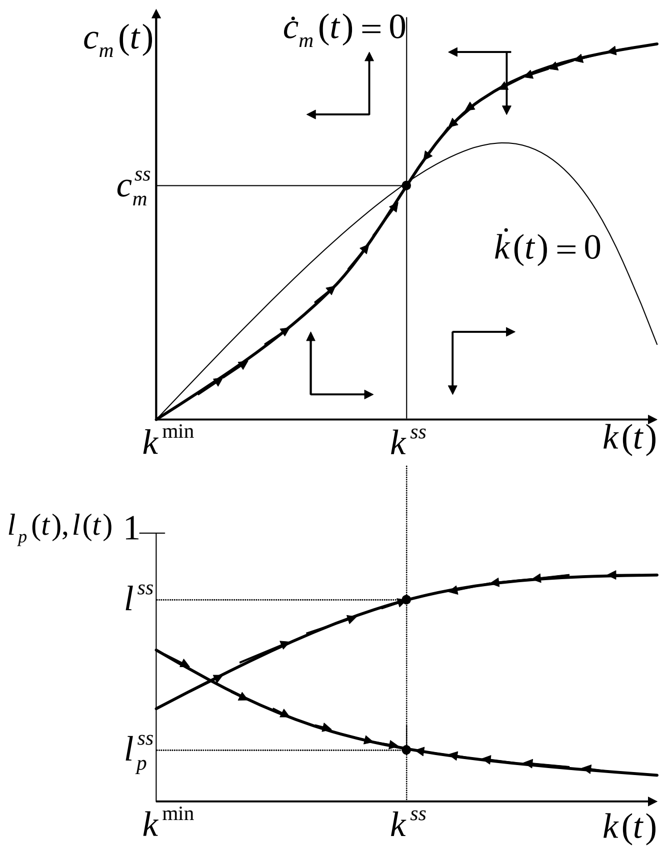

Figure 1: Dynamic behavior

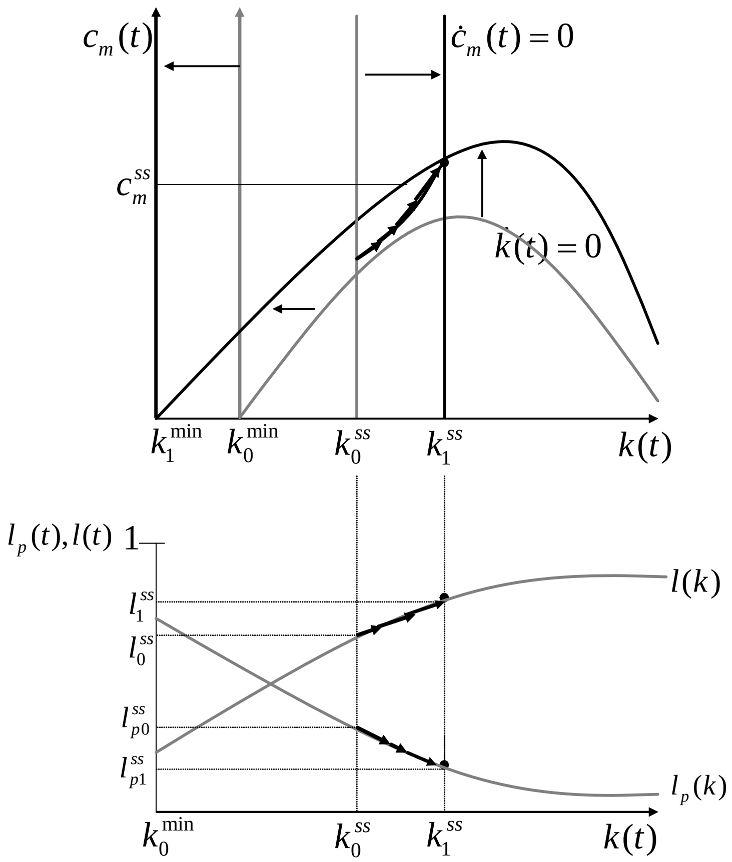

Figure 2: Dynamic effect of an improvement in the agricultural technology

can use the same phase diagram in Figure 2 to display the dynamic effect that such technological change generates. When there is an improvement in the technology of the manufacturing sector, the production and the marginal productivity of capital in this sector rise. As a consequence, the $\dot{k}$ locus goes up and the $\dot{c}$ locus moves to the right. The economy goes towards a new steady state with a higher level of capital, a lower portion of labor devoted to predation and a higher portion of labor and capital devoted to the manufacturing sector. Throughout the transition there is also an *amplification effect* due to the fall in predation: when per capita capital goes up, the portion of labor that goes to agriculture goes down, increasing the labor share and reducing the amount of labor devoted to predation. This reduction in predation and the increase in productive labor amplify the effect of technological change on capital accumulation. Furthermore, the fall in predation has a direct positive effect on the portion of the return on capital that goes to savers, encouraging additional capital accumulation.

9. The role of institutions

To analyze the role of institutions in the model we make a straightforward extension. We introduce a parameter, ξ, in the predation technology which reduces the productivity of predation, and we interpret such parameter as an index of institutional quality. To be more precise, consider that the amount of income obtained through predation is equal to $g\left(l_p, \xi\right) \widetilde{y}(t)$, with $g(., \xi)$ having the same properties defined above but now $g(.)$ is a strictly decreasing function of ξ, the index of institutional quality. Figure 3 displays the dynamic effect of an improvement in institutional quality, ξ. The effect of an improvement in institutional quality is similar to the improvements in the technology of productive sectors analyzed above. In this case, the improvement in institutional quality reduces the productivity of predation, encouraging individuals to devote more resources to productive sectors. Since, the economy have reached the minimum amount of consumption in agriculture, workers that were allocated in the predation are now flowing to manufacturing. Graphically, the reallocation of workers at the moment of change in ξ implies a jump down in predation curve, $l_p(k)$, and a jump up in productive labor curve, $l(k)$, as we can observe at bottom part of the Figure 3. As a consequence, the production and the marginal productivity of capital in manufacturing rise, implying that the $\dot{k}$ locus goes up and the $\dot{c}$ locus moves to the right. The economy goes towards a new steady state with a higher level of capital, a lower portion of labor devoted to predation and a higher portion of labor and capital devoted to the manufacturing sector. Finally, as the cases of technological improvements in the productive sectors, the *amplification effect* appears: when per capita capital goes up, the portion of labor that goes to agriculture goes down, increasing the labor share and reducing the amount of labor devoted to predation. This reduction in predation and the increase in productive labor amplify the effect of the improvement on the institutional quality on capital accumulation. Secondly, the fall in predation which has a direct positive effect on the portion of the return on capital that goes to savers generate more additional incentives to expand furthermore the capital accumulation.

10. Conclusions

This paper presents a neoclassical growth model with predation in which firms produce agricultural and manufactured goods. A country faces the typical "food problem", it has to satisfy first its subsistence needs of agricultural goods before starting to consume manufactured goods. As the country accumulates capital and subsistence needs begin to be satisfied, a structural change occurs: labor is reallocated from agriculture to manufacturing, which implies a higher weight of manufacturing in the added value of the economy. Due to the fact that agriculture is less labor intensive than manufacturing, the structural change implies that (aggregate) labor share rises during the transition when the initial per capita capital is lower than the steady state level. This increase in the labor share implies a reduction in incentives to predate and a reallocation of labor from predation to production. Thus, this paper analyzes how predation affects capital accumulation and also how capital accumulation affects predation and the resulting feedback process.

This paper also contributes to understand differences in per capita income among countries. Despite many authors have identified differences in productivity as one of the the main factors accounting for differences in per capita income, these differences in productivity are not empirically high enough to generate the differences that are observed in per capita income. This paper proposes a mechanism that amplifies the differences in per capita income generated by differences in productivity. It is wellknown from the literature on economic growth that when productivity rises, there is a direct standard effect on production and an indirect standard effect due to the accumulation of capital: the rise in productivity increases the return on savings and thus, the incentives to accumulate more capital. However, in our model, there exists an additional mechanism related to the

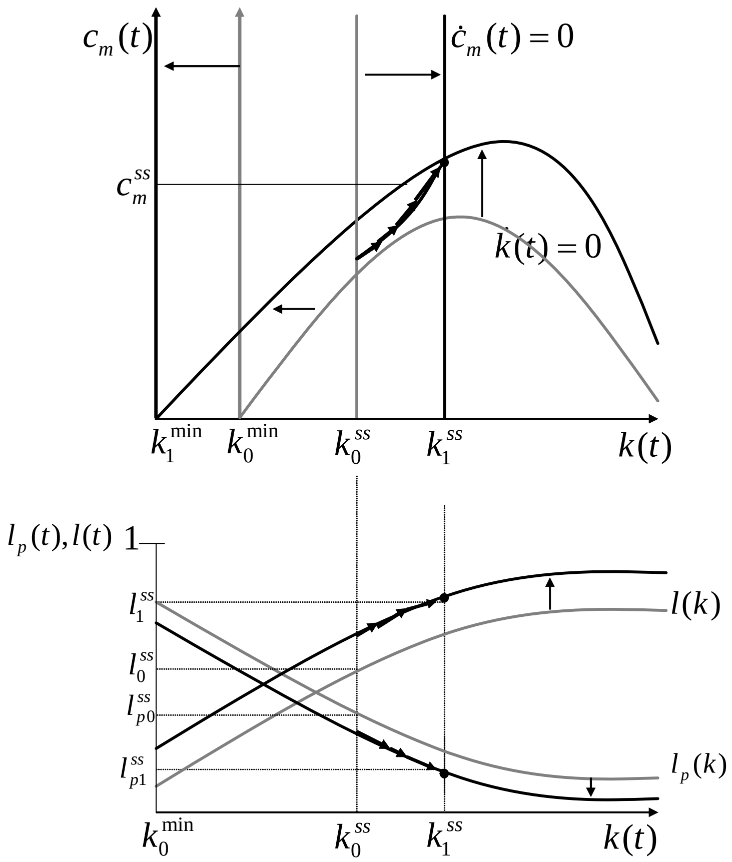

Figure 3: Dynamic effect of an improvement in the institutional quality

reallocation of resources among sectors and the reduction in predation: when productivity rises, the per capita capital rises, generating a reallocation of resources from agriculture to manufacturing and so, an increase in labor share, which reduces the incentive for predation and increases the portion of labor devoted to production. This increase in the amount of labor devoted to production has three effects: first, a direct effect on per capita production; second, an indirect effect due to the accumulation of capital: when labor rises, it increases the marginal productivity of capital and thus, the incentive to accumulate more capital; thrid, the reduction in the portion of labor devoted to predation implies that the share of the marginal product of capital that goes to savers increases, raising the return on savings and promoting further the accumulation of capital.

Finally, we analyze the role of institutional quality in reducing predation and in the reallocation of productive factors among sectors. We study the case of an institutional improvement that that reduces the productivity of the predation technology. Such a change discourages predation by increasing the share of labor devoted to production. This increase in the labor allocated in the productive sector implies an increase in the production of manufacturing but it also encourages the accumulation of capital due to two mechanisms: (i) it increases the marginal product of capital and therefore the return on savings; (ii) it reduces the portion of the payments to capital that goes to predation, increasing the return on savings as well. Furthermore, insofar the economy accumulates capital and the share of manufacturing over total income is increasing with respect to agriculture, the total labor share increases, and this promotes the reallocation of labor from predation to production even more.

11. References

Acemoglu, D. (1995) "Reward Structures and the Allocation of Talent", *European Economic Review*, January, 39 (1), 17-33.

Acemoglu, D., Johnson, S. and J.A. Robinson (2005) "Institutions as a Fundamental Cause of Long-Run", in Handbook of Economic Growth, Volume 1, Part 1, ed. Philippe Aghion and Steve Durlauf, Elsevier.

Acemoglu, D., and T. Verdier (1998) "Property rights, corruption and the allocation of talent: a general equilibrium approach", *Economic Journal*, 108, 1381-1403.

Anderson, D.A. (1999) "The Aggregate Burden of Crime", *Journal of Law and Economics*, 42 (2), 611-642.

Andonova, V and H. Zuleta (2009) "Beyond Moral Hazard: The Effect of Firm-Level Compensation Strategies on Economic Conflict", *Peace Economics, Peace Science and Public Policy*, 15 (1), article 5.

Banerjee, A. and E. Duflo (2007) "The economic lives of the poor. Journal of Economic Perspectives", 21 (1), 141-167.

Bethencourt, C. and F. Perera-Tallo (2014) "Declining Predation during Development: a Feedback Process", *Economica*, forthcoming.

Bourguignon, F. (1999) "Crime as a social cost of poverty and inequality: a review focusing on developing countries", Washington, D.C.-París, World Bank-Delta, mimeo.

Brunetti, A. and B.Weder (2003) "A Free Press is Bad News for Corruption", *Journal of Public Economics*, 87 (7-8), 1801-1824.

Chassang, S. and S. Padró-i-Miquel (2009) "Conflict and Development", *American Economic Review*, 99 (2), 282-286.

Chassang, S. and G. Padró-i-Miquel (2010) "Savings and Predation", *Journal of the European Economic Association*, 8 (2-3), 645-654.

Córdoba, J.C. and M. Ripoll (2009) "Agriculture and aggregation", *Economics Letters*, 105 (1), 110-112.

Dal Bó, E. and P. Dal Bó (2011) "Workers, Warriors, and Criminals: Social Conflict in General Equilibrium", *Journal of the European Economic Association*, 9(4), 646-677.

Decreuse, B. and P. Maarek (2009) "Foreign direct investment and the labor share in developing countries. A theory and some evidence", GREQAM, Universite d' Aix-Marseille, mimeo.

Easterly, W. and R. Levine (2001) "It's Not Factor Accumulation: Stylized

Facts and Growth Models", *World Bank Economic Review*, 15 (2), 177-219.

Echevarria, C. (1998) "A Three Factor Agricultural Production Function: The Case of Canada", *lInternational Economic Review*, 12 (3), 63-75.

Gollin, D. (2002) "Getting Income Shares Right", *Journal of Political Economy*, 110 (2), 458-474.

Gollin, D., Parente, S. and R. Rogerson (2002) "The Role of Agriculture in Development", *American Economic Review*, 92 (2), 160-164.

Gollin, D., Parente, S. and R. Rogerson (2004) "Farm Work, Home Work, and International Productivity Differences", *Review of Economic Dynamics*, 7 (4), 827-850.

Gollin, D., Parente, S. and R. Rogerson (2007) "The food problem and the evolution of international income levels", *Journal of Monetary Economics*, 54 (4), 1230-1255.

Grossman, H.I. and M. Kim (1996) "Inequality, predation, and welfare", NBER Working Paper no. 5704.

Grossman, H.I. and M. Kim (2002) "Predation, Efficiency, and Inequality", *Journal of Institutional and Theoretical Economics*, 158 (3), 393-407.

Hall, R.E. and C.I. Jones (1999) "Why do Some Countries Produce so much more Output per Worker than Others", *Quarterly Journal of Economics*, 114 (1), 83-116.

Harrison, A.E. (2005) "Has globalization eroded labor's share? Some cross country evidence", University of California Berkeley, mimeo.

Khwaja, A. and A. Mian (2004) "Unchecked intermediaries: price manipulation in an emerging stock market", *Journal of Financial Economics*, 78 (1), 203-41.

Londoño, J. and R. Guerrero (1998) "Violencia en América Latina: epidemiología y costos", in Asalto al desarrollo: violencia en América Latina, eds. J. Londoño, A. Gaviria y R. Guerrero, Washington, D.C., Banco InterAmericano de Desarrollo.

Maarek, P. (2010) "Labor share, Informal sector and Development", GREQAM, Universite d'Aix-Marseille, mimeo.

Morck, R., Yeung, B. and W. Yu (2000) "The information content of stock markets. Why do emerging markets have synchronous stock price movements?", *Journal of Financial Economics*, 58 (1/2), 215-260.

Mel, S., McKenzie, D. and C. Woodruff (2008) "Who are the microenterprise owners? Evidence from Sri Lanka", The World Bank, Policy Research Working Paper Series No. 4635.

Murphy, K.M., Shleifer, A. and R.W. Vishny (1991) "The Allocation of Talent: Implications for Growth", *Quarterly Journal of Economics*, May, 106 (2), 503-530.

Murphy, K.M., Shleifer, A. and R.W. Vishny (1993) "Why Is Rent-Seeking So Costly to Growth?", *American Economic Review*, 83 (2), 409-414.

Narita, R. (2011) "Self Employment in Developing Countries: a Search-Equilibrium Approach", mimeo.

Paldam, M. (2001) "Corruption and Religion: Adding to the Economic Model", *Kyklos*, 54 (2/3), 383-414.

Paldam, M. (2002) "The Cross-Country Pattern of Corruption: Economics, Culture and the Seesaw Dynamics", *European Journal of Political Economy*, 18 (2), 215-240.

Parente, S.L. and E.C. Prescott (1993) "Changes in the Wealth of the Nations", *Federal Reserve Bank of Minneapolis Quarterly Review*, 17 (2), 3-16.

Quah, D. (1996) "Twin Peaks: Growth and Convergence in Models of Distribution Dynamics", *Economic Journal*, 106 (437), 1045-1055.

Quah, D. (1997) "Empirics for Growth and Distribution: Stratification, Polarization and Convergence Clubs", *Journal of Economic Growth*, 2 (1), 27-59.

Restuccia, D., Yang, D.T. and X. Zhu (2008) "Agriculture and aggregate productivity: A quantitative cross-country analysis", *Journal of Monetary Economics*, 55 (2), 234-250.

Ortega, D. and F. Rodriguez (2006) "Are capital shares higher in poor countries? Evidence from Industrial Surveys", Wesleyan Economics, working paper No. 2006-023.

Rehman , H.U. and A. Naveed (2007) "Determinants of Corruption and its Relation to GDP: (A Panel study)", *Journal of Political Studies*, 41 (4), 27-59.

Schrag, J. and S. Scotchmer (1993) "The Self-Reinforcing Nature of Crime", Working Paper 93-11, *Center for the Study of Law and Society*, School of Law, University of California, Berkeley.

Soares, R. and J. Naritomi (2010) "Understanding High Crime Rates in Latin America: The Role of Social and Policy Factors", in: Rafael Di Tella, Sebastian Edwards, and Ernesto Schargrodsky (editors). The Economics of Crime: Lessons for and from Latin America, University of Chicago Press, 19-55.

Schrag, J. and S. Scotchmer (1993) "The Self-Reinforcing Nature of Crime", Working Paper 93-11, Center for the Study of Law and Society, School of Law, University of California, Berkeley.

Temkin, B. (2009) "Informal Self-Employment in Developing Countries: Entrepreneurship or Survivalist Strategy? Some Implications for Public Policy", *Analyses of Social Issues and Public Policy*, 9 (1), 135-156.

Tornell, A. and P.R. Lane (1999) "The Voracity Effect", *American Economic Review*, 89 (1), 22-46.

Treisman, D. (2000) "The Causes of Corruption: A Cross-National Study", *Journal of Public Economics*, 76 (3), 399-457.

Valentinyi, A. and B. Herrendorf (2008) "Measuring Factor Income Shares at the Sector Level", *Review of Economic Dynamics*, 11 (4), 820-835.

Zuleta, H. (2004) "Persistence and Expansion of Guerrilla Activities: the case of Colombia", *Journal of Income Distribution,* 13 (1-2),105-115.

12. Appendix

Proof of Lemma 3

The labor share in the productive sector is defined by:

$$\lambda = \frac{wl}{y} = \frac{wl}{p_a y_a + y_m} = \frac{wl}{\frac{wl_a}{1-\alpha-\beta} + \frac{wl_m}{1-\alpha}} = \frac{(1-\alpha)(1-\alpha-\beta)}{(1-\alpha)\frac{l_a}{l} + (1-\alpha-\beta)\frac{l_m}{l}}$$

$$\lambda = \frac{(1-\alpha)(1-\alpha-\beta)}{(1-\alpha)\psi_a + (1-\alpha-\beta)(1-\psi_a)} = \frac{(1-\alpha)(1-\alpha-\beta)}{(1-\alpha-\beta)+\beta\psi_a} \tag{19}$$

where $\psi_a \equiv \frac{l_a}{l}$ is the portion of productive labor used in agriculture. We used in the second equality equations (14), (17), (5) and (6). It is straightforward to see that the labor share decreasing function of the portion of productive labor devoted to agriculture. ■

Proof of Lemma 4

Using equation (10) and the fact that all household are identical ($\widetilde{l_p} = l_p$), it follows that:

$$\phi(l_p) = \frac{g'(l_p)(1-l_p)}{[1-g(l_p)]} = \lambda \tag{20}$$

where $\phi : [0,1] \rightarrow \Re_+$.

It was assumed that $g(1) < 1$ which implies:

$$\phi(1) = \frac{g'(1)(1-1)}{[1-g(1)]} = 0 \tag{21}$$

By assumption $g'(0) \geq 1$ and $g(0) = 0$, which imply that:

$$\phi(0) = \frac{g'(0)(1-0)}{[1-g(0)]} = g'(0) \geq 1 \tag{22}$$

Note that if $l_p < 1$ and $\phi(l_p) \leq 1$ then

$$\phi'(l_p) = \frac{g"(l_p)(1-l_p)-g'(l_p)+\phi(l_p)g'(l_p)}{[1-g(l_p)]} \leq \frac{g"(l_p)(1-l_p)-g'(l_p)+g'(l_p)}{[1-g(l_p)]} = \frac{g"(l_p)(1-l_p)}{[1-g(l_p)]} < 0 \tag{23}$$

It follows from equations (21) and (22) and, the fact that $\phi(l_p)$ is continuous and strictly decreasing when $\phi(l_p) \leq 1$ (see equation 23), that there is a unique $l_p^{\min} \in [0,1)$, such that $\phi\left(l_p^{\min}\right) = 1$, being $l_p^{\min} = 0$ when $g'(0) = 1$. Furthermore, it follows from equation (21) and definition of $l_p^{\min}$, that $\phi\left(l_p^{\min}\right) > 1$ when $l_p < l_p^{\min}$.

Finally, it follows from equation (23) and definition of $l_p^{\min}$ that $\phi\left(l_p^{\min}\right)$ is strictly decreasing when $l_p \in \left[l_p^{\min}, 1\right]$. ■

Proof of Proposition 5

The factors and agriculture goods markets clearing conditions are the following:

$$
\begin{aligned}
l_a + l_m &= l && (24)\\
k_a + k_m &= k && (25)\\
\overline{c} &= y_a && (26)
\end{aligned}
$$

Using (14), (15), (17),(18) we get:

$$
\frac{(1-\alpha-\beta)}{\alpha}\frac{k_a}{l_a} = \frac{(1-\alpha)}{\alpha}\frac{k_m}{l_m} \tag{27}
$$

Using (24), (25) and (27):

$$
\begin{aligned}
(1-\alpha-\beta)\frac{k_a}{l_a} &= (1-\alpha)\frac{k-k_a}{l-l_a} \Leftrightarrow\\
(1-\alpha-\beta)k_a l - (1-\alpha)k_a l_a + \beta k_a l_a &= (1-\alpha)k l_a - (1-\alpha)k_a l_a\\
(1-\alpha-\beta)k_a l + \beta k_a l_a &= (1-\alpha)k l_a\\
k_a &= \frac{(1-\alpha)k l_a}{(1-\alpha-\beta)l + \beta l_a} = \frac{(1-\alpha)\psi_a}{(1-\alpha-\beta)+\beta\psi_a}k && (28)
\end{aligned}
$$

Using (5), (26) and (28):

$$
\begin{aligned}
\overline{c} &= \Gamma_a z^\beta \left(\frac{(1-\alpha)\psi_a}{(1-\alpha-\beta)+\beta\psi_a}\right)^\alpha \psi_a^{1-\alpha-\beta} k^\alpha l^{1-\alpha-\beta} \Leftrightarrow\\
\left(\frac{(1-\alpha)\psi_a}{(1-\alpha-\beta)+\beta\psi_a}\right)^\alpha \psi_a^{1-\alpha-\beta} &= \frac{\overline{c}}{\Gamma_a z^\beta k^\alpha l^{1-\alpha-\beta}} && (29)
\end{aligned}
$$

Using (19), (20) and (5), it yields the following equation system:

$$
\begin{aligned}
\phi(1-l) - \frac{(1-\alpha)(1-\alpha-\beta)}{(1-\alpha-\beta)+\beta\psi_a} &= 0 && (30)\\
\left(\frac{(1-\alpha)}{(1-\alpha-\beta)+\beta\psi_a}\right)^\alpha \psi_a^{1-\beta} l^{1-\alpha-\beta} - \Phi &= 0 && (31)
\end{aligned}
$$

where $\Phi \equiv \frac{\overline{c}}{\Gamma_a z^\beta k^\alpha}$. Treating l and ψ_a as the endogenous variables and using the Implicit Function Theorem and the Cramer rule:

$$\frac{\partial l}{\partial \Phi} = -\frac{\begin{vmatrix} 0 & \frac{\beta}{[(1-\alpha-\beta)+\beta\psi_a]}\lambda \\ -1 & \Phi\left[-\frac{\alpha\beta}{(1-\alpha-\beta)+\beta\psi_a} + \frac{(1-\beta)}{\psi_a}\right] \end{vmatrix}}{\begin{vmatrix} -\phi'(1-l) & \frac{\beta}{[(1-\alpha-\beta)+\beta\psi_a]}\lambda \\ (1-\alpha-\beta)\frac{\Phi}{l} & \Phi\left[-\frac{\alpha\beta}{(1-\alpha-\beta)+\beta\psi_a} + \frac{(1-\beta)}{\psi_a}\right] \end{vmatrix}} =$$

$$-\frac{\frac{\beta}{[(1-\alpha-\beta)+\beta\psi_a]}\lambda}{\frac{\Phi\lambda^2}{1-\alpha}\frac{1}{l}\left[-\frac{\phi'(1-l)l}{\phi(1-l)}\frac{(1-\alpha)(1-\alpha-\beta)}{\lambda} + \beta + \alpha\right]} < 0 \text{ *****VER FOOTNOTE 1*****}$$

$$\frac{\partial \psi_a}{\partial \Phi} = -\frac{\begin{vmatrix} -\phi'(1-l) & 0 \\ (1-\alpha-\beta)\frac{\Phi}{l} & -1 \end{vmatrix}}{\begin{vmatrix} -\phi'(1-l) & \frac{\beta}{[(1-\alpha-\beta)+\beta\psi_a]}\lambda \\ (1-\alpha-\beta)\frac{\Phi}{l} & \Phi\left[-\frac{\alpha\beta}{(1-\alpha-\beta)+\beta\psi_a} + \frac{(1-\beta)}{\psi_a}\right] \end{vmatrix}} =$$

$$-\frac{\phi'(1-l)}{\frac{\Phi\lambda^2}{l(1-\alpha)}\left[-\frac{\phi'(1-l)l}{\phi(1-l)}\left[\frac{(1-\beta)}{\psi_a}+\beta\right]-\beta\right]} > 0 \text{ *****VER FOOTNOTE 2*****}$$

where the denominator, $-\frac{\phi'(1-l)l}{\phi(1-l)}\left[\frac{(1-\beta)}{\psi_a}+\beta\right]-\beta$, is always positive since:

$$\begin{aligned}
\phi(l_p) &= \frac{g'(l_p)(1-l_p)}{[1-g(l_p)]} \\
\frac{\partial \ln \phi(l_p)}{\partial l_p} &= \frac{\phi'(l_p)}{\phi(l_p)} = \frac{g''(l_p)}{g'(l_p)} - \frac{1}{(1-l_p)} - \frac{g'(l_p)}{1-g(l_p)} \\
-\frac{\phi'(1-l)l}{\phi(1-l)} &= -\frac{g''(l_p)(1-l_p)}{g'(l_p)} + 1 + \phi(l_p) > 1 + \lambda
\end{aligned}$$

and in the last inequality we use the assumption that $g(l_p)$ is concave. Thus,

$$\begin{aligned}
\frac{\partial l}{\partial k} &= \frac{\partial l}{\partial \Phi}\frac{\partial \Phi}{\partial k} > 0; \quad \frac{\partial l}{\partial z} = \frac{\partial l}{\partial \Phi}\frac{\partial \Phi}{\partial z} > 0; \quad \frac{\partial l}{\partial \Gamma_a} = \frac{\partial l}{\partial \Phi}\frac{\partial \Phi}{\partial \Gamma_a} > 0; \quad \frac{\partial l}{\partial \overline{c}} = \frac{\partial l}{\partial \overline{c}}\frac{\partial \overline{c}}{\partial \Gamma_a} < 0 \\
\frac{\partial \psi_a}{\partial k} &= \frac{\partial \psi_a}{\partial \Phi}\frac{\partial \Phi}{\partial k} < 0; \quad \frac{\partial \psi_a}{\partial z} = \frac{\partial \psi_a}{\partial \Phi}\frac{\partial \Phi}{\partial z} < 0; \quad \frac{\partial \psi_a}{\partial \Gamma_a} = \frac{\partial \psi_a}{\partial \Phi}\frac{\partial \Phi}{\partial \Gamma_a} < 0; \quad \frac{\partial \psi_a}{\partial \overline{c}} = \frac{\partial \psi_a}{\partial \overline{c}}\frac{\partial \overline{c}}{\partial \Gamma_a} > 0 \\
\frac{\partial l_p}{\partial k} &= -\frac{\partial l}{\partial \Phi}\frac{\partial \Phi}{\partial k} < 0; \quad \frac{\partial l_p}{\partial z} = -\frac{\partial l}{\partial \Phi}\frac{\partial \Phi}{\partial z} < 0; \quad \frac{\partial l_p}{\partial \Gamma_a} = -\frac{\partial l}{\partial \Phi}\frac{\partial \Phi}{\partial \Gamma_a} < 0; \quad \frac{\partial l_p}{\partial \overline{c}} = -\frac{\partial l}{\partial \overline{c}}\frac{\partial \overline{c}}{\partial \Gamma_a} > 0
\end{aligned}$$

■
67

12.1. Dynamic System

It follows from (18), (24), (25) and (28) that:

$$k_m = k - k_a = \frac{(1-\alpha-\beta)(1-\psi_a)}{(1-\alpha-\beta)+\beta\psi_a}k \tag{32}$$

$$l_m = (1-\psi_a)l \tag{33}$$

$$y_m = \Gamma_m \left[\frac{(1-\alpha-\beta)}{(1-\alpha-\beta)+\beta\psi_a}\right]^{\alpha}(1-\psi_a)k^{\alpha}l^{1-\alpha} \tag{34}$$

$$\delta + r = \alpha\frac{y_m}{k_m} = \alpha\Gamma_m\left[\frac{(1-\alpha-\beta)+\beta\psi_a}{(1-\alpha-\beta)}\right]^{1-\alpha}\left(\frac{l}{k}\right)^{1-\alpha} \tag{35}$$

Thus, it follows from the above equations and the capital accumulation equation (2) and the Euler equation (11) that:

$$\begin{aligned}\dot{k}(t) &= y_m\,(k(t)) - c_m(t) - \delta k(t)\\ \frac{\dot{c_m}(t)}{c_m(t)} &= (r\,(k(t))+\delta)\,(1-g\,(l_p\,(k(t)))) - \delta - \rho\end{aligned}$$

---

[6]**********************

FOOTNOTE 1
****

NO VEO ESTE DENORMINADOR DENTRO DEL CORCHETE, NO ME SALE NI DE COÑA.

ADEMAS SE SUPONE QUE TIENE QUE SER IGUAL QUE EL DE ABAJO Y TAMPOCO VEO COMO
****
****

[7]**********************

FOOTNOTE 2
****

NO VEO ESTE DENORMINADOR DENTRO DEL CORCHETE, NO ME SALE NI DE COÑA.

ADEMAS SE SUPONE QUE TIENE QUE SER IGUAL QUE EL DE ARRIBA Y TAMPOCO VEO COMO
****
****

where

$$y_m(k) = \Gamma_m \left[\frac{(1-\alpha-\beta)}{(1-\alpha-\beta)+\beta\psi_a(k)}\right]^{\alpha} (1-\psi_a(k))\, k^{\alpha}\,[l(k)]^{1-\alpha} \tag{36}$$

$$r(k) = \alpha\Gamma_m \left[\frac{(1-\alpha-\beta)+\beta\psi_a(k)}{(1-\alpha-\beta)}\right]^{1-\alpha} \left(\frac{l(k)}{k}\right)^{1-\alpha} - \delta \tag{37}$$

Using equation (29) it is possible to rewrite the above functions as follows:

$$y_m(k) = \Gamma_m \left[\frac{1\text{-}\alpha\text{-}\beta}{1\text{-}\alpha}\right]^{\alpha} \left[\frac{\overline{c}}{\Gamma_a z^{\beta}}\right] \frac{(1\text{-}\psi_a(k))}{\psi_a^{1-\beta}} l(\psi_a(k))^{\beta} \text{ ****VER FOOTNOTE 3 *****} \tag{38}$$

$$r(k) = \alpha\Gamma_m \left[\frac{1\text{-}\alpha}{1\text{-}\alpha\text{-}\beta}\right]^{1-\alpha} \left[\frac{\psi_a(k)^{1-\beta}\,\Gamma_a z^{\beta} l(\psi_a(k))^{1-\beta}}{\overline{c}}\right]^{\frac{1-\alpha}{\alpha}} \text{-}\delta \text{ ****VER FOOTNOTE 4*****} \tag{39}$$

[8][9]

Proof of Lemma 6

It follows straightforward from (38) that y_m is an increasing function of k. It follows from equation (39) that r is a decreasing function in k when $h \equiv \psi_a l$ is a decreasing function of k. To prove that, it is enough to prove that h is an increasing function of $\Phi \equiv \frac{\overline{c}}{\Gamma_a z^{\beta} k^{\alpha}}$. We may rewrite the equation system (30) and

---

[8] **********************

FOOTNOTE 3
****

NO VEO DE DONDE SALE $l(\psi_a(k))$, ESA FUNCIÓN NO ESTÁ DEFINIDA EN NINGUNA PARTE
****
****

[9] **********************

FOOTNOTE 4
****

NO VEO DE DONDE SALE $l(\psi_a(k))$, ESA FUNCIÓN NO ESTÁ DEFINIDA EN NINGUNA PARTE
****
****

(31) as follows:

$$\phi(1-l) - \frac{(1-\alpha)(1-\alpha-\beta)}{(1-\alpha-\beta)+\beta\frac{h}{l}} = 0$$

$$\left(\frac{\phi(1-l)}{(1-\alpha-\beta)}\right)^{\alpha} h^{1-\beta} l^{-\alpha} - \Phi = 0$$

Using the Cramer rule:

$$\frac{\partial h}{\partial \Phi} = -\frac{\begin{vmatrix} \left[-\frac{\phi'(1-l)l}{\phi(1-l)} - \frac{\beta}{(1-\alpha-\beta)+\beta\frac{h}{l}}\frac{h}{l}\right]\phi(1-l)l & 0 \\ \alpha\Phi\left[\frac{\phi'(1-l)l}{\phi(1-l)} - 1\right]l & -1 \end{vmatrix}}{\begin{vmatrix} \left[-\frac{\phi'(1-l)l}{\phi(1-l)} - \frac{\beta\lambda}{(1-\alpha)(1-\alpha-\beta)}\frac{h}{l}\right]\phi(1-l)l & \frac{\beta\phi(1-l)}{(1-\alpha-\beta)+\beta\frac{h}{l}}\frac{1}{l} \\ \alpha\Phi\left[\frac{\phi'(1-l)l}{\phi(1-l)} - 1\right]l & (1-\beta)\frac{\Phi}{h} \end{vmatrix}} =$$

$$\frac{\overbrace{\left[-\frac{\phi'(1-l)l}{\phi(1-l)} - \frac{\beta}{(1-\alpha-\beta)+\beta\frac{h}{l}}\frac{h}{l}\right]\phi(1-l)l}^{\oplus}}{\underbrace{\left[-\frac{\phi'(1-l)l}{\phi(1-l)} - \frac{\beta}{(1-\alpha-\beta)+\beta\frac{h}{l}}\frac{h}{l}\right]\phi(1\text{-}l)l(1-\beta)\frac{\Phi}{h} - \frac{\beta\phi(1\text{-}l)}{(1\text{-}\alpha\text{-}\beta)+\beta\frac{h}{l}}\frac{\alpha\Phi}{l}\underbrace{\left[\frac{\phi'(1\text{-}l)l}{\phi(1\text{-}l)}\text{-}1\right]}_{\ominus}l}_{\oplus}} > 0$$

where we have used the fact that $-\frac{\phi'(1-l)l}{\phi(1-l)} - \frac{\beta}{(1-\alpha-\beta)+\beta\frac{h}{l}}\frac{h}{l} > 0$:

$$-\frac{\phi'(1-l)l}{\phi(1-l)} - \frac{\beta}{(1-\alpha-\beta)+\beta\frac{h}{l}}\frac{h}{l} = -\frac{\phi'(1-l)l}{\phi(1-l)} - \left[1 - \frac{(1-\alpha-\beta)}{(1-\alpha-\beta)+\beta l_a}\right]$$

Given that $-\frac{\phi'(1-l)l}{\phi(1-l)} > 1+\lambda$ (see Proof of Proposition **5**), we obtain

$$\begin{aligned} -\frac{\phi'(1-l)l}{\phi(1-l)} - \left[1 - \frac{(1-\alpha-\beta)}{(1-\alpha-\beta)+\beta l_a}\right] &> \\ 1+\lambda - \left[1 - \frac{(1-\alpha-\beta)}{1-\alpha}\right] &= 1+\lambda - \left[\frac{\beta}{1-\alpha}\right] > \lambda > 0 \end{aligned}$$

Therefore, it follows from equation (39) that:

$$\frac{\partial r}{\partial k} = \underbrace{\frac{\partial r}{\partial h}}_{\oplus}\underbrace{\frac{\partial h}{\partial \Phi}}_{\oplus}\underbrace{\frac{\partial \Phi}{\partial k}}_{\ominus} < 0$$

Finally, the effect of a change in ψ_a on the interest rate after predation results as follows:

$$\frac{\frac{\partial[(r(\psi_a)+\delta)(1-g(l_p(\psi_a)))]}{\partial\psi_a}}{(r(\psi_a)+\delta)(1-g(l_p(\psi_a)))} = \frac{\frac{\partial(r(\psi_a)+\delta)}{\partial\psi_a}}{(r(\psi_a)+\delta)} + \frac{\frac{\partial(1-g(l_p(\psi_a)))}{\partial\psi_a}}{(1-g(l_p(\psi_a)))} =$$

$$\frac{(1-\alpha)(1-\beta)}{\alpha}\left[\frac{1}{\psi_a} - \frac{\phi(1-l)}{-\phi'(1-l)l}\frac{\beta}{(1-\alpha-\beta)+\beta\psi_a}\right] - \frac{g'(l_p)}{(1-g(l_p))}\frac{\partial l_p}{\partial\lambda}\frac{\partial\lambda}{\partial\psi_a} = \text{***VER FOO}$$

$$\frac{(1-\alpha)(1-\beta)}{\alpha}\left[\frac{1}{\psi_a} - \frac{\phi(1-l)}{-\phi'(1-l)l}\frac{\beta}{(1-\alpha-\beta)+\beta\psi_a}\right] - \frac{g'(l_p)}{(1-g(l_p))}\frac{1}{-\phi'(1-l)}\frac{\beta\lambda}{(1-\alpha-\beta)+\beta\psi_a} =$$

$$\frac{(1-\alpha)(1-\beta)}{\alpha}\left[\frac{1}{\psi_a} - \frac{\phi(1-l)}{-\phi'(1-l)l}\frac{\beta}{(1-\alpha-\beta)+\beta\psi_a}\right] - \frac{g'(l_p)l}{(1-g(l_p))}\frac{1}{-\phi'(1-l)l}\frac{\beta\lambda}{(1-\alpha-\beta)+\beta\psi_a} =$$

$$\frac{(1-\alpha)(1-\beta)}{\alpha}\left[\frac{1}{\psi_a} - \frac{\phi(1-l)}{-\phi'(1-l)l}\frac{\beta}{(1-\alpha-\beta)+\beta\psi_a}\right] - \frac{\phi(1-l)}{-\phi'(1-l)l}\frac{\beta\lambda}{(1-\alpha-\beta)+\beta\psi_a} =$$

$$\frac{(1-\alpha)(1-\beta)}{\alpha}\left[\frac{1}{\psi_a} - \frac{\phi(1-l)}{-\phi'(1-l)l}\frac{\beta\left[1+\frac{\lambda\alpha}{(1-\alpha)(1-\beta}\right]}{(1-\alpha-\beta)+\beta\psi_a}\right] >$$

$$\frac{(1-\alpha)(1-\beta)}{\alpha}\left[\frac{1}{\psi_a} - \frac{\frac{\beta\left[1+\frac{\lambda\alpha}{(1-\alpha)(1-\beta)}\right]}{1+\lambda}}{(1-\alpha-\beta)+\beta\psi_a}\right] >$$

$$\frac{(1-\alpha)(1-\beta)}{\alpha}\left[\frac{1-\alpha-\frac{\beta\left[1+\frac{\lambda\alpha}{(1-\alpha)(1-\beta)}\right]}{1+\lambda}}{(1-\alpha-\beta)+\beta\psi_a}\right]$$

where we have used $-\frac{\phi'(1-l)l}{\phi(1-l)} > 1+\lambda$ and the assumption that $\lambda_a > \alpha(1-\beta) \Leftrightarrow (1-\alpha)(1-\beta) = \lambda_a + \alpha\beta > \alpha$. Thus:

$$\frac{\partial\left[(r(\psi_a)+\delta)(1-g(l_p(\psi_a)))\right]}{\partial k} = \frac{\partial\left[(r(\psi_a)+\delta)(1-g(l_p(\psi_a)))\right]}{\partial\psi_a}\frac{\partial\psi_a}{\partial k} < 0$$

where $\frac{\partial\psi_a}{\partial k} < 0$ by Proposition **5.** ■
[10]

10 **********************

FOOTNOTE 5

****

NO VEO DE DONDE SALE EL CORCHETE DE ESA ECUACION, SEGURO QUE ESTA BIEN, PERO YO NO LO VEO

****

****

Proof of Lemma 7

Let's define $\underline{l}$ as the amount of labor at equilibrium which would imply that the labor share of the economy were equal to the one in agriculture, and $\overline{l}$ the amount of labor at equilibrium which would imply the labor share of the economy were equal to the one in manufacture:

$$\begin{aligned}\underline{l} \overset{def}{\Leftrightarrow} \phi(\underline{l}) &= 1-\alpha-\beta \\ \overline{l} \overset{def}{\Leftrightarrow} \phi(\overline{l}) &= 1-\alpha\end{aligned}$$

Obviously, $l \in \left(\underline{l}, \overline{l}\right)$. Let's define $\underline{k}$ as the level of capital required to produce the subsistence level of consumption when $l = \underline{l}$):

$$\underline{k} \overset{def}{\Leftrightarrow} \overline{c} = \Gamma_a z^\beta \underline{k}^\alpha \underline{l}^{1-\alpha-\beta} \Leftrightarrow \underline{k} = \left(\frac{x}{\underline{l}^{1-\alpha-\beta}}\right)^{\frac{1}{\alpha}}$$

where $x = \frac{\overline{c}}{\Gamma_a z^\beta}$. It follows from the definitions of $\underline{k}$ and $\underline{l}$ that:

$$\psi_a\left(\underline{k}\right) = 1$$

Let's define $\overline{k}$ as the level of capital which implies that the per capita income growth rate is zero when all resources of the economy are devoted to the manufacturing sector (i.e., there is neither production in the agriculture nor predation in the economy):

$$\overline{k} \overset{def}{\Leftrightarrow} \delta\overline{k} = \Gamma_m \overline{k}^\alpha \Leftrightarrow \overline{k} = \left(\frac{\Gamma_m}{\delta}\right)^{\frac{1}{1-\alpha}}$$

If $\underline{k} < \overline{k}$ it is possible to define the following function:

$$f\left(x\right) = \max_{k\in\left[\underline{k},\overline{k}\right]} \frac{y_m(k)}{k}$$

It follows from the definition of $\overline{k}$ that:

$$f\left(\left(\left(\frac{\Gamma_m}{\delta}\right)^{\frac{\alpha}{1-\alpha}} \underline{l}^{1-\alpha-\beta}\right)\right) = \max_{k\in\left[\left(\frac{\Gamma_m}{\delta}\right)^{\frac{1}{1-\alpha}},\overline{k}\right]} \frac{y_m(k)}{k} = \max_{k\in\left[\overline{k},\overline{k}\right]} \frac{y_m(k)}{k} = \frac{y_m(\overline{k})}{\overline{k}} < \frac{\Gamma_m\overline{k}^\alpha}{\overline{k}} = \delta$$

$$\begin{aligned}\lim_{x\to0} f\left(x\right) &= \lim_{x\to0}\max_{k\in\left[0,\overline{k}\right]} \frac{y_m(k)}{k} = \lim_{x\to0}\max_{k\in\left[0,\overline{k}\right]} \frac{\Gamma_m\left[\frac{(1-\alpha-\beta)}{(1-\alpha-\beta)+\beta\psi_a(k)}\right]^\alpha\left(1-\psi_a\left(k\right)\right)\left[l\left(\psi_a\left(k\right)\right)\right]^{1-\alpha}}{k^{1-\alpha}} = \\ \sup_{k\in\left[0,\overline{k}\right]} \Gamma_m\left(\frac{\overline{l}}{k}\right)^{1-\alpha} &= +\infty\end{aligned}$$

It follows from equations (36) and (29) that $\frac{y_m(k)}{k}$ is a decreasing function of x :[11]

$$\frac{\partial\left(\frac{y_m(k)}{k}\right)}{\partial x}=\underbrace{\frac{\partial\left(\frac{y_m(k)}{k}\right)}{\partial \psi_a}}_{\ominus}\underbrace{\frac{\partial \psi_a}{\partial x}}_{\oplus}<0$$

Thus, it follows from the Maximum and the Envelope Theorem that there is a unique $x_1 \in \left(0, \left(\frac{\Gamma_m}{\delta}\right)^{\frac{\alpha}{1-\alpha}} \underline{l}^{1-\alpha-\beta}\right)$ such that $f(x_1)=\delta$ and $\forall x<x_1\ f(x_1)>\delta$. Thus, if $x<x_1$,it is possible to define $k^{\min}$ and $k^{\max}$ such that:

$$k^{\min} \overset{def}{\Leftrightarrow} 0 = y_m\left(k^{\min}\right)-\delta k^{\min} \Leftrightarrow y_m\left(k^{\min}\right) = \delta k^{\min} \tag{40}$$

$$k^{\max} \overset{def}{\Leftrightarrow} k^{\max} = \min\left\{k\in\left[k^{\min},\overline{k}\right] \text{ s.th. } y_m(k) = \delta k\right\} \tag{41}$$

It follows from (29), (37) and (40) that:

$$\lim_{x\to 0}\psi_a(k) = 0 \Rightarrow \lim_{x\to 0} y_m(k) = \Gamma_m k^{\alpha}\overline{l}^{1-\alpha} \Rightarrow \lim_{x\to 0} k^{\min} = 0 \Rightarrow$$
$$\lim_{x\to 0} r\left(k^{\min}\right) = \lim_{k\to 0} r\left(k\right) =$$
$$\lim_{k\to 0}\alpha\Gamma_m\left[\frac{(1-\alpha-\beta)+\beta\psi_a(k)}{(1-\alpha-\beta)}\right]^{1-\alpha}\left(\frac{l(k)}{k}\right)^{1-\alpha}-\delta \geq \lim_{k\to 0}\alpha\Gamma_m\left(\frac{\underline{l}}{k}\right)^{1-\alpha}-\delta = +\infty$$

If $r\left(k^{\min}\right)\geq\rho+\delta$ when $x=x_1$, then $\Omega=x_1$. If $r\left(k^{\min}\right)<\rho+\delta$ when $x=x_1$, then, there is x_2 such that if $x=x_2$ then $r(k^{\min})=\rho+\delta$. Thus, in this case $\Omega=x_2$. ■
[12]

[11] **********************

FOOTNOTE 6
****

NO VEO QUE DE ESAS ECUACIONES SALGA ESE RESULTADO, NO VEO DE DONDE
****
****

[12] **********************

FOOTNOTE 7
****

FERNI QUERIDO, ESA PRUEBA ESTÁ BIEN? REALMENTE NO DEMOSTRAMOS LO QUE ESTÁ EN EL LEMA 7
****
****