

Growth and Aid: a Hump-Shaped Relationship*

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February 22, 2016

Abstract

Empirical findings show that the effect of aid on growth is ambiguous. We build a growth model in which the government receives aid to finance a productive public good and agents devote time to appropriate public resources. International aid increases public resources, raising the provision of the productive public good but, in turn, promoting rent-seeking. Consequently, the relationship between aid and growth appears hump-shaped: too much aid is counterproductive for growth, particularly when institutions are weak. International aid transmits the growth of the donor to the receptor country, but an excessive amount of it may harm income convergence and even prevent convergence among ex-ante identical countries.

Keywords: Growth Theory, Foreign Aid, Convergence.

JEL code: F35, O10, O41

*We thank the participants at seminars in Universidad de La Laguna for helpful comments. Both authors are members of CAERP and they wish to acknowledge its support and stimulating environment. We thank the Spanish Ministry of Science and Technology for Grant ECO2013-48884-C3-3-P for financial support. Finally, we are also grateful to José-Víctor Ríos-Rull and Pierre Pestieau for helpful comments.

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1. Introduction

“Does international aid have positive effects on growth?” The literature on growth and aid does not offer any conclusive answer, with empirical evidence often being ambiguous and mixed¹. The subsequent question is “Is there any explanation for this ambiguity?”. Empirical literature has devoted considerable effort to answering this question. There is a group of papers that evidence that foreign aid stimulates the emergence of rent-seeking activities by powerful social groups in order to appropriate resources from the government². Another set of papers finds document that aid may erode the quality of institutions³, which would have negative consequences on growth⁴ **[[In contrast to the empirical literature, the theoretical literature on the link between growth, governance and aid is scarce. As Temple (2010) states, the majority of studies have typically omitted considerations such as governance and political economy that are key in the debate about aid effectiveness.]]** This aim of this paper is to fill this gap in the literature by presenting a growth model which sheds some light on the empirical facts mentioned above. The paper focuses on the effect of international aid on growth, development and convergence among countries, providing useful conclusions for aid policies.

This paper presents a model in which agents devote time to work and to rent-seeking activities in order to appropriate public resources from the government. There are two sources of public resources: tax collection obtained from non distortionary taxes and the foreign aid. The amount of public revenue that is not grabbed by rent-seeking activities is used to finance a public good, which is productive *à la* Barro and generates endogenous growth.

We show that in spite of the presence of non-distortionary taxes, a hump-shaped relationship exists between aid and growth. This result is due to three offsetting mechanisms: *(i)* an increase in aid raises government’s resources and so the provision of public good, expanding the productivity of the private sector and thus, increasing the growth rate, *(ii)* the increase in the government’s resources

¹See detailed revisions of the literature by Clemens et al. (2011) and Minoiu and Reddy (2010).

²**[[See Reinnika and Svensson (2004) and Maren (1997) and, more recently, Ravalion (2014). This last paper argues that though external aid is targeted to poor people, it goes through a government that does not share that goal. This explains why the purpose of aid has been thwarted at times.]]**

³See Rajan and Subramanian (2007) and Djankov et al. (2008).

⁴See Djankov et al. (2008) for an excellent discussion of the literature.

improves the profitability of rent-seeking activities, which reduces the amount of resources devoted to the public good, *(iii)* since agents devote more time to rent-seeking activities, labor supply drops, thus reducing growth. We show that for low levels of aid the first effect prevails, while for high levels the other two effects are stronger. Thus, international aid is not always beneficial for the growth rate of the receptor country. We show that an “optimal” level of international aid exists which maximizes the growth rate of the receptor country. If international aid is above this level, aid becomes harmful to growth.

Institutions play a key role: an improvement in the quality of institutions affects growth positively in two different ways. First, it reduces the reward for rent-seeking activities, raising the amount of time devoted to productive activities and so, increasing the growth rate. Second, it reduces the negative effect that international aid has on promoting rent-seeking (the “rent-seeking promoting effect”). Thus, improvement in the quality of institution increases the optimal level international aid that maximizes the growth rate. This means that countries with bad institutions are not only the ones that grow slower but are also the ones in which international aid is more likely to have a counterproductive effect on growth, since the “rent-seeking promoting effect” of international aid is stronger among them.

We modify the model to analyze the effect of international aid on the convergence across countries. To do this we first consider a *two-different countries* model. In this model there are two different countries, the *North* and the *South*, the *North* being the country that would grow faster in the absence of international aid and the donor of international aid. We show that the existence of international aid permits the transmission of the growth of the donor country, the *North*, to the receptor country, the *South*, raising the growth rate in the *South* to the same rate as in the *North*. Nevertheless, the *North* and the *South* might not converge in income levels. We show that an excessive international aid may undermine the convergence of the *South* to the *North* due to the fact that international aid promotes rent-seeking. More precisely, there are two balanced growth paths in the model: the “good” balanced growth path and the “bad” one. The convergence rate, defined as the ratio between income of the *South* and income of the *North*, is higher on the good balanced growth path than on the bad balanced growth path. If the *South* is on the bad balanced growth path, a reduction of international aid from the *North* during the transition would make the *South* converge on the good balanced growth path. Thus, a drop in aid may even promote the convergence of the per capita income of the receptor country to that of the donor.

The transitional dynamic to the balanced growth path is characterized by a decline in the time devoted to rent-seeking when the initial per capita income of the receptor country is smaller than the value on the balanced growth path. The reason for that is that along the transition the reward for working, the wage, increases faster than the reward for rent-seeking activities, tax collection plus international aid. The transitional dynamic may be characterized by multiple equilibria. The convergence to the bad balanced growth path may be easily avoided by reducing the level of international aid below a certain threshold.

Finally, after studying the *two-different* countries model in which the receptor country would grow slower than the donor country in absence of international aid, we analyze the case of *two-alike* countries in which countries are identical in all aspects except for their initial amounts of capital and in which the richer country donates international aid to the poorer one. We show that in this case the existence of international aid may prevent the convergence of both countries to the same balanced growth path. The reason for that is that international aid promotes rent-seeking in the receptor country, generating an “asymmetric” balanced growth path where there is no convergence in per capita income and, consequently, international aid from the donor to the receptor country self-perpetuates. Along this asymmetric balanced growth path, the level of rent-seeking in the receptor country is higher than on the symmetric one due to international aid. Furthermore, the donor country diverts part of its public resources from productive public investment to international aid. As a consequence, the growth rate on the asymmetric balanced growth path is lower than the growth rate of the “symmetric” balanced growth path, in which countries converge in per capita income and, consequently, there is no need for international aid. Thus, in the asymmetric balanced growth path, aid results harmful for both growth and convergence.

The theoretical literature that links aid with growth uses the neoclassical capital accumulation framework and assumes that aid is distributed among agents as transfers. There are two approaches; the first one considers the Ramsey-Cass-Koopmans model (see Obstfeld, 1999[[, **Scholl, 2009, and Arellano et al., 2009**]]). In this environment, an increase in aid raises the per capita consumption of receptor country without affecting its steady state per capita capital. An increase of aid in the infinite-horizon household model represents a pure wealth effect which increases the consumption in the present and in the future, without having any substitutive effect, that is, without affecting the reward for saving. So, aid does not affect per capita capital at the steady state. The second approach considers the Diamond’s model (see Dalgaard, Hansen and Tarp, 2004). In this

framework, the effect of aid on per capita capital is not well determined since transfers in the first period of life encourage saving and capital accumulation, while transfers in the second period of life have just the opposite effect. **[[Our view is different from these previous contributions since, we focus on the most plausible case in which aid is not transferred to consumers but used for public expenditure projects intended to increase the productivity of the economy. Thus, this paper can be framed in the recent literature that links aid with growth and that considers the role for aid-financed public investment or infrastructure (see, among others, Agénor and Yilmaz, 2013, Chatterjee et al., 2003 and Chatterjee and Turnovsky, 2004, 2005). However, in contrast with existent papers, we introduce a novel element: we analyze the incentive problems that aid may produce on devoting resources to rent-seeking. Both, the rent-seeking mechanism and the productive role of the aid generate the hump-shaped relationship between growth and aid in our model.]]**

Many papers have studied the effect of rent-seeking activities in the economy (see Bethencourt and Perera-Tallo, 2015, for a review), however, to the best of our knowledge, only the paper by Svensson (2000) has analyzed the impact of these activities in an international aid setting. Svensson (2000) poses a repeated game model in which different groups interact strategically to capture the aid received by the government. Nevertheless, he does not analyze the consequences of this activity on growth, which is the goal of the present paper. There are many other aspects in the literature on external aid which are not so closely linked with our model, like political economy issues, transaction costs or agency problems. For an excellent survey see Paul (2006).

The rest of this paper is organized as follows. Section 2 presents the basic model where individuals devote time to rent-seeking and to work in the market and, the government uses tax collection and international aid to finance a productive good. Section 3 analyzes the behavior and decisions of agents in the long run and characterizes the balanced growth path. Section 4 studies the income convergence of two different countries: the *North*, the one with larger growth rate in the absence of convergence and donor of international aid, and the *South*, the receptor country. Section 5 analyzes the effect of international aid on the income convergence in an alternative model with two alike countries. Finally, Section 6 concludes. All the technical details and proof are included in the Appendix.

2. The Model

Time is continuous with an infinite horizon. Population, $N(t)$, is constant. There is a single good in the economy that can be used for consumption, investment and as a public good provided by the government:

$$y(t) = c(t) + g(t) + \dot{k}(t) + \delta k(t) \quad (2.1)$$

where y denotes per capita production, c denotes per capita consumption, g denotes per capita public good provided by the government, k denotes per capita capital, and $\delta \in (0, 1)$ denotes the depreciation rate.

2.1. Preferences

There is a continuum of identical households indexed in $[0, 1]$ with preferences given by a time separable utility function:

$$\int_0^\infty u(c(t))e^{-\rho t} dt \quad (2.2)$$

where $\rho > 0$ is the discount rate of the utility function, c denotes the consumption, and $u(\cdot)$ is the isoelastic felicity function:

$$u(c) = \begin{cases} \frac{c^{1-\sigma}}{1-\sigma} & \text{if } \sigma \in (0, 1) \cup (1, +\infty) \\ \ln(c) & \text{if } \sigma = 1 \end{cases} \quad (2.3)$$

2.2. Production technology

The technology is given by the following production function:

$$y(t) = Ag(t)^{1-\alpha}K(t)^\alpha L(t)^{1-\alpha} \quad (2.4)$$

where K denotes capital, L denotes labor, g denotes the per capita amount of a public good provided by the government and $A > 0$.

2.3. Fiscal policy

The government obtains resources from two different sources: financial funds from international aid, denoted by $aid(t)$ (in per capita terms) and tax collection coming

from non distortionary lump-sum taxes that are calculated as a portion (τ) of the per capita income. We consider lump-sum taxes to exclude standard effects of distortionary taxes from the analysis. Since our paper focuses on the implications of rent-seeking, we prefer to introduce non-distortionary taxes to make it clear that the mechanisms that we analyze in the model are related to rent-seeking, and not to the distortionary effects of taxes which do not play any role in this paper. Thus:

$$T(t) = \tau y(t) + aid(t) \quad (2.5)$$

where T denotes the per capita revenues and $\tau \in (0, 1)$ is the fixed tax rate.

We denote the ratio international aid-(national) income by $a(t) \equiv aid(t)/y(t)$. Thus, the ratio government revenues-income is as follows:

$$\frac{T(t)}{y(t)} = \tau + a(t) \quad (2.6)$$

((Rent-seeking activities are associated with government revenues. This means that not all government revenues are devoted to financing the public good, a part of them are “transferred” to agents)) **[[In this economy not all government revenues are devoted to financing the public good⁵. Since agents may be involved in rent-seeking activities, a part of public resources are “transferred” to agents⁶.]]** The amount of income transfers that each agent “obtains” depends on the rent-seeking effort that the agent makes to get the transfer. Each individual is endowed with one unit of time each period and decides the portion of time devoted to rent-seeking activities, $l_p^i(t)$, and the portion devoted

⁵[[As aid and corruption are two key issues for developing economies, many empirical papers have investigated the relation between both dimensions. Although the studies are not consensual, most of them tends to show that aid enhances corruption (Knack, 2001, Alesina and Weder, 2002). The reason is that aid can give bad incentives for recipient countries to reduce the need for governments to collect taxes and to encourage rent-seeking activities.]]

⁶[[Actually, public officers (bureaucrats) who manage the international aid are the group of agents that can appropriate resources from the government trough corruption. However, in our representative agent model, we are considering that the representative agent may devotes a fraction of her time to rent-seeking. Alternatively, we might assume that a fraction of the households’ members are corrupted officers who are involved in rent-seeking activities.]]

to work, $1 - l_p^i(t)$. The transfer that agent i receives, $tr^i(t)$, is as follows:

$$tr^i(t) = \frac{(l_p^i(t))^\theta}{\int_0^1 (l_p^j(t))^\theta dj} Bl_p(t)^\beta T(t)$$

where $l_p(t) = \int_0^1 l_p^j(t) dj$ denotes the per capita time devoted to rent-seeking activities in the economy, $B \in [0, 1]$, $\beta \in (0, 1)$ and $\theta \in (0, 1)$. Thus, the **[[total]]** portion of government revenues appropriated by rent-seeking activities, Bl_p^β , is an increasing function of the per capita rent-seeking effort **[[; and the share]]** ((. The)) share of per capita rent-seeking income received by agent i , $(l_p^i(t))^\theta / \int_0^1 (l_p^j(t))^\theta dj$, increases with the agent's rent-seeking effort, l_p^i , and decreases with other agents' effort, l_p^j . The parameter B is as an index of the productivity of rent-seeking technology, which may be interpreted as an index of institutional weakness.

The government uses the part of government revenues that is not grabbed by rent-seeking activities to provide the public good:

$$g(t) = T(t) [1 - Bl_p(t)^\beta] \quad (2.7)$$

2.4. Firms

Firms maximize profits:

$$\max_{K(t), L(t)} Ag(t)^{1-\alpha} K(t)^\alpha L(t)^{1-\alpha} - w(t)L(t) - (\delta + r(t))K(t)$$

where w and r denote the prices of both labor and physical capital respectively. The first order conditions are standard ones:

$$(1 - \alpha) \frac{Ag(t)^{1-\alpha} K(t)^\alpha L(t)^{1-\alpha}}{L(t)} = w(t) \quad (2.8)$$

$$\alpha \frac{Ag(t)^{1-\alpha} K(t)^\alpha L(t)^{1-\alpha}}{K(t)} = (\delta + r(t)) \quad (2.9)$$

Using the labor market clearing condition $L(t) = (1 - l_p(t))N(t)$ in the above equations it yields:

$$(1 - \alpha) \frac{y(t)}{(1 - l_p(t))} = w(t) \quad (2.10)$$

$$\alpha \frac{y(t)}{k(t)} = (\delta + r(t)) \quad (2.11)$$

2.5. Households

Households face the following optimization problem:

$$\begin{aligned} & \underset{c(t), l_p^i(t)}{Max} \int_{t=0}^{\infty} u(c(t)) e^{-\rho t} dt & (2.12) \\ \text{s.t. } & r(t)b(t) + w(t)(1 - l_p^i(t)) + \frac{(l_p^i(t))^\theta}{\int_0^1 (l_p^j(t))^\theta dj} B l_p(t)^\beta T(t) - \tau y(t) = \dot{b}(t) + c(t) \end{aligned}$$

where $b(t)$ denotes the household assets at time t , that is, the household wealth, and $1 - l_p^i(t)$ denotes the amount of time devoted to work in the labor market. The first order conditions associated with the household's optimization problem (2.12) are the following:

$$w(t) = \frac{\theta B l_p(t)^\beta T(t)}{(l_p^i(t))^{1-\theta} \int_0^1 (l_p^j(t))^\theta dj} \quad (2.13)$$

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma} (r(t) - \rho) \quad (2.14)$$

$$\lim_{t \rightarrow \infty} e^{-\rho t} c(t)^{-\sigma} b(t) = 0 \quad (2.15)$$

Equation (2.13) means that the marginal income from working should be equal to the marginal income from rent seeking. The second equation (2.14) is the typical Euler equation and the last one (2.15) is the transversality condition. Given that all agents are alike in equilibrium, the time devoted to rent-seeking activity, l_p^i , is the same for all agents, $l_p^i(t) = l_p^j(t) = l_p(t) \forall i, j$. This symmetry condition together with equation (2.10) and equation (2.13) imply:

$$\frac{l_p^{1-\beta}}{1 - l_p} = \frac{\theta B}{(1 - \alpha)} \frac{T}{y} \quad (2.16)$$

Using the Implicit Function Theorem, we define $l_p(\cdot)$ as a function that relates the time devoted to rent-seeking activities with the ratio government revenues-income, $\frac{T}{y}$, the productivity of the rent-seeking technology (or index of institutional weakness), B , and the labor share, $(1 - \alpha)$:

$$l_p \left(\frac{T}{y}, B, (1 - \alpha) \right) \stackrel{Def}{\iff} \frac{\left(l_p \left(\frac{T}{y}, B, (1 - \alpha) \right) \right)^{1-\beta}}{1 - l_p \left(\frac{T}{y}, B, (1 - \alpha) \right)} = \frac{\theta B}{(1 - \alpha)} \frac{T}{y} \quad (2.17)$$

where

$$\frac{\partial l_p \left(\frac{T}{y}, B, (1 - \alpha) \right)}{\partial \left(\frac{T}{y} \right)} = \frac{\theta B}{(1 - \alpha)} \frac{[1 - l_p]^2 l_p^\beta}{(1 - \beta) [1 - l_p] + l_p} > 0 \quad (2.18)$$

$$\frac{\partial l_p \left(\frac{T}{y}, B, (1 - \alpha) \right)}{\partial B} = \frac{\theta}{(1 - \alpha)} \frac{T}{y} \frac{[1 - l_p]^2 l_p^\beta}{(1 - \beta) [1 - l_p] + l_p} > 0 \quad (2.19)$$

$$\frac{\partial l_p \left(\frac{T}{y}, B, (1 - \alpha) \right)}{\partial (1 - \alpha)} = - \frac{\theta B}{(1 - \alpha)^2} \frac{T}{y} \frac{[1 - l_p]^2 l_p^\beta}{(1 - \beta) [1 - l_p] + l_p} < 0 \quad (2.20)$$

Equations (2.18), (2.19) and (2.20) show that when either the ratio government revenues-income, $\frac{T}{y}$, or the index of institutional weakness, B , increase, rent-seeking becomes more profitable, encouraging agents to devote more time to rent-seeking activities. To the contrary, a rise in the labor share, $(1 - \alpha)$, increases the return of working in the market and so, the opportunity cost of devoting time to rent-seeking, thus discouraging rent-seeking. In order to make the notation more compact, we will re-write the function $l_p \left(\frac{T}{y}, B, (1 - \alpha) \right)$ as $l_p \left(\frac{T}{y} \right)$ when this does not cause confusion.

3. Balanced growth path

Using equations (2.4), (2.11), (2.14) and (2.16) we obtain the growth rate of the economy, denoted by v , which is constant when the ratio international aid-income, a , is constant:

$$v = \frac{\dot{c}}{c} = \frac{1}{\sigma} \left(\alpha A^{\frac{1}{\alpha}} \left[\frac{(1 - \alpha)}{\theta B} \left(l_p \left(\frac{T}{y} \right) \right)^{1 - \beta} \left[1 - B \left(l_p \left(\frac{T}{y} \right) \right)^\beta \right] \right]^{\frac{1 - \alpha}{\alpha}} - \delta - \rho \right) \quad (3.1)$$

Assuming that $B > (1 - \beta)^{\frac{1}{\beta}}$, the growth rate reaches its maximum level at the following value (see appendix 7.1 for details):

⁷If $B \leq (1 - \beta)^{\frac{1}{\beta}}$, the rent-seeking technology is not productive enough to offset the benefits of international aid. In this case, the model would behave as Barro's model with international aid: more international aid generates more growth.

$$\left(\frac{T}{y}\right)^* = \tau + a = \frac{(1-\alpha)}{\theta} \frac{(1-\beta)^{\frac{1-\beta}{\beta}}}{(B)^{\frac{1}{\beta}} - (1-\beta)^{\frac{1}{\beta}}} \quad (3.2)$$

Thus, the relationship between the growth rate and the ratio government revenues-income shows a hump-shaped form: first, it is strictly increasing, and then, strictly decreasing (see Figure 3.1). Assuming that $\tau < \left(\frac{T}{y}\right)^*$, this implies that the level of international aid over income that maximizes the growth rate, a^* , is as follows:

$$a^* = \frac{(1-\alpha)}{\theta} \frac{(1-\beta)^{\frac{1-\beta}{\beta}}}{(B)^{\frac{1}{\beta}} - (1-\beta)^{\frac{1}{\beta}}} - \tau \quad (3.3)$$

Notice that the reason for this hump-shaped relation is different from Barro's model (Barro, 1990). In Barro's model, the hump-shaped relationship between growth and tax rate is due to the distortionary effect that the income tax has on the present-future consumption decision. Moreover, the effect of international aid on growth would be always positive, since the increase in aid would imply an increase of government's expenditure for the same tax rate. The distortionary tax effect of Barro's model has been erased from our model to make it clear that this distortion does not play any role. In our model, the effect of an increase in government revenue produces three different effects. First, it increases, at least before rent-seeking occurs, government expenditure on the public good, which increases the productivity of the private sector and the growth rate (that is the standard effect as in Barro's model). Second, the increase in government revenues encourages rent-seeking and thus, reduces the portion of government revenues that are devoted to productive government expenditure, reducing growth. Third, since agents devote more effort to rent-seeking activities, the labor supply goes down, reducing growth. When the amount of international aid is below level a^* , the first effect prevails and international aid has a positive effect on growth. When international aid exceeds level a^* , negative effects prevail and international aid hampers growth. Thus, an increase in international aid does not always promote growth. Despite aid contributing to financing a productive public good, it also encourages rent-seeking. In fact, if international aid is above threshold a^* , an increase in international aid becomes harmful for growth.

Figure 3.1 represents the effect of an improvement in the quality of institutions, that is, a reduction in institutional weakness, B , on growth (see appendix 7.1 for technical details). An improvement in institutional quality reduces the incentive

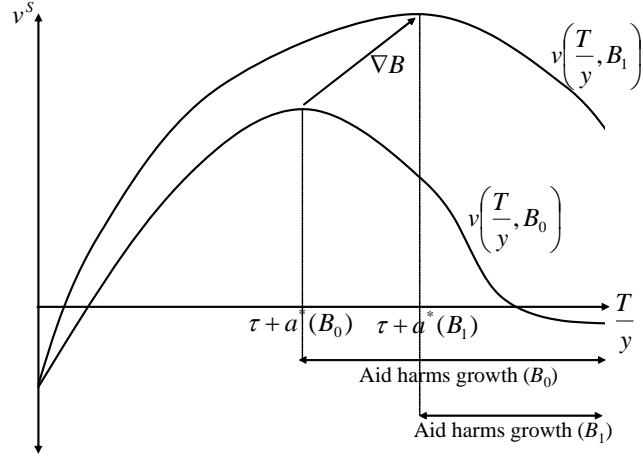


Figure 3.1: Effect of an improvement in institutions ($B_0 > B_1$)

to devote time to rent-seeking, increasing the share of government revenues devoted to financing the productive public good and the portion of time devoted to work in the market. As a result, it has a positive effect on the growth rate. Furthermore, an improvement in institutional quality reduces “the rent-seeking promoting effect” of international aid, raising the “optimal” level of international aid that maximizes the growth rate, a^* (see equation 3.3 and figure 3.1). A possible reinterpretation of this result of the model is that, given an amount of aid a , there is a certain threshold level of institutional weakness, $B(a)$, such that the effect of aid on growth is positive if the institutional weakness is lower than this threshold level, $B(a)$:

$$\frac{\partial v(\tau + a, B)}{\partial a} \geq 0 \Leftrightarrow B \leq B(a)$$

where $B(a)$ is defined as follows:

$$B(a) \stackrel{def}{\Leftrightarrow} a = \frac{(1-\alpha)}{\theta} \frac{(1-\beta)^{\frac{1-\beta}{\beta}}}{(B(a))^{\frac{1}{\beta}} - (1-\beta)^{\frac{1}{\beta}}} - \tau \Leftrightarrow$$

$$B(a) = (1-\beta) \left[\frac{(1-\alpha)}{\theta(1-\beta)(a+\tau)} + 1 \right]^\beta$$

Summarizing, countries with the worst institutions not only are the ones that grow slower, they are also the ones in which international aid is more likely to be counterproductive⁸.

4. Convergence

In this section, we study the convergence of the country that receives international aid, which we call *South*, to the country that gives it, which we call *North*. We will analyze how international aid affects the convergence in per capita income.

4.1. Balanced Growth Path

We assume that: (i) in absence of international aid, the *North* grows faster than the *South*; (ii) the growth rate in the *North* is smaller than the maximum growth rate of the *South*; (iii) if all the labor force in the *South* is devoted to rent-seeking,

⁸[[This is consistent with empirical findings in the literature which evidences that the institutional quality of the donor's government is a key factor to explain aid effectiveness. See Ravallion (2014) for a detailed discussion.]]

then the growth rate in the *South* is smaller than in the *North*⁹:

$$v^N > \frac{1}{\sigma} \left(\alpha A^{\frac{1}{\alpha}} \left[\frac{(1-\alpha)}{\theta B} (l_p(\tau))^{1-\beta} \left[1 - B (l_p(\tau))^\beta \right] \right]^{\frac{1-\alpha}{\alpha}} - \delta - \rho \right) = v_{\text{no aid}}^S \quad (4.1)$$

$$v^N < \frac{1}{\sigma} \left(\alpha A^{\frac{1}{\alpha}} \left[\frac{(1-\alpha)}{\theta B^{\frac{1}{\beta}}} (1-\beta)^{\frac{1-\beta}{\beta}} \beta \right]^{\frac{1-\alpha}{\alpha}} - \delta - \rho \right) = \max_{\frac{T}{y}} v^S \quad (4.2)$$

$$v^N > \lim_{l_p \rightarrow 1} \frac{1}{\sigma} \left(\alpha A^{\frac{1}{\alpha}} \left[\frac{(1-\alpha)}{\theta B} (l_p)^{1-\beta} \left[1 - B (l_p)^\beta \right] \right]^{\frac{1-\alpha}{\alpha}} - \delta - \rho \right) = \frac{1}{\sigma} \left(\alpha A^{\frac{1}{\alpha}} \left[\frac{(1-\alpha)}{\theta B} [1-B] \right]^{\frac{1-\alpha}{\alpha}} - \delta - \rho \right) \quad (4.3)$$

where superscripts N and S mean, respectively, *North* and *South*. A possible reason for the *North* growing faster than the *South* is that the *North* may have better institutions (a lower B) than the *South*. Even in the case that the *North* and the *South* have the same technology and preferences, a difference in institutional quality (in the parameter B) would generate differences in growth rates.

We consider that the *North* spends the fraction ψ of its income on international aid to the *South*:

$$a = \frac{\text{aid}}{y^S(t)} = \psi \frac{y^N(t)}{y^S(t)} = \frac{\psi}{\tilde{y}(t)}$$

where $\tilde{y}(t) = y^S(t)/y^N(t)$ denotes the convergence index. Thus, along the balanced growth path (BGP from now on) the convergence index remains constant, $\tilde{y}(t) = \tilde{y} \forall t$, which implies that both countries are growing at the same rate:

$$v^N = \frac{1}{\sigma} \left(\alpha A^{\frac{1}{\alpha}} \left[\frac{(1-\alpha)}{\theta B} \left(l_p \left(\tau + \frac{\psi}{\tilde{y}} \right) \right)^{1-\beta} \left[1 - B \left(l_p \left(\tau + \frac{\psi}{\tilde{y}} \right) \right)^\beta \right] \right]^{\frac{1-\alpha}{\alpha}} - \delta - \rho \right)$$

As Figure 4.1 shows, the hump-shaped relationship between growth and international aid in the *South* implies the existence of two BGPs, in one of them the ratio international aid-income is lower than the optimal level, and in the other it is the opposite. Notice that the convergence index is higher in the BGP with the

⁹Obviously, this can never happen in autarchy. However, when there is international aid and the ratio international aid-income in the *South* goes to infinity, the amount of per capita labor devoted to rent-seeking goes to one.

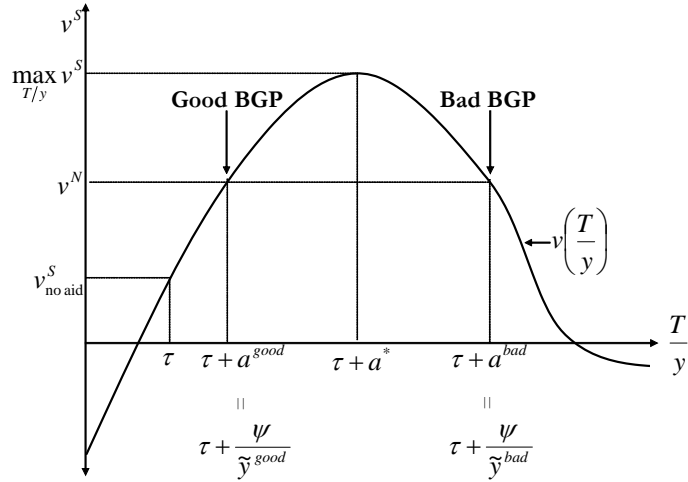


Figure 4.1: Multiple BGP

lower ratio international aid-income than in the other one. We will call from now on “good” BGP the BGP with the higher convergence index and “bad” BGP the other:

$$\tilde{y}^{good} > \tilde{y}^{bad} \Rightarrow \frac{\psi}{\tilde{y}^{good}} = a^{good} < a^* < a^{bad} = \frac{\psi}{\tilde{y}^{bad}}$$

A simple way to make the *South* economy converge to the good BGP (the one with the higher convergence index) would consist of reducing the amount of international aid which the *South* receives. More precisely, if international aid is bounded above by the growth maximizing international aid level, a^* , that is,

$$a(t) = \min \left\{ \frac{\psi}{\tilde{y}}, a^* \right\}$$

the bad BGP disappears, since $a^* < a^{bad}$, the *South* will converge to the good BGP, where the convergence index is the largest. We will study the dynamic to the BGP in the next subsection.

4.2. Transitional dynamic to the Balanced Growth Path

We will analyze the transitional dynamic to the BGP in two cases:

1. When international aid that the *North* provides to the *South* is proportional to the income of the *North*:

$$aid(t) = \psi y^N(t) \Leftrightarrow a(t) = \frac{aid(t)}{y^S(t)} = \psi \frac{y^N(t)}{y^S(t)} = \frac{\psi}{\tilde{y}(t)} \quad (4.4)$$

2. When international aid that the *North* provides to the *South* is proportional to the income of the *North*, but it is bounded by the amount of international aid that maximizes the consumption growth rate (and the return on capital):

$$a(t) = \min \left\{ \frac{\psi}{\tilde{y}(t)}, a^* \right\} \Leftrightarrow aid(t) = \min \{ \psi y^N(t), a^* y^S(t) \}$$

4.2.1. International aid proportional to the income of the *North*

Let's define $\tilde{y}(t) = y^S(t)/y^N(t)$, $\tilde{c}(t) = c^S(t)/y^N(t)$, $\tilde{k}(t) = k^S(t)/y^N(t)$. The dynamic system which describes the behavior of the economy in the *South* in the case that the received aid is proportional to the income of the *North* would be as follows (see appendix 7.2 for technical details):

$$\frac{\dot{\tilde{c}}(t)}{\tilde{c}(t)} = \frac{1}{\sigma} \left(\alpha \frac{\tilde{y}(\tilde{k}(t))}{\tilde{k}(t)} - \delta - \rho \right) - v^N \quad (4.5)$$

$$\dot{\tilde{k}}(t) = \tilde{y}(\tilde{k}(t)) (1-\tau) + B \left(\hat{l}_p(\tilde{k}(t)) \right)^\beta \left[\tau \tilde{y}(\tilde{k}(t)) + \psi \right] - (\delta + v^N) \tilde{k}(t) - \tilde{c}(t) \quad (4.6)$$

where:

$$\tilde{y}(\tilde{k}(t)) = A^{\frac{1}{\alpha}} \left[\frac{(1-\alpha)}{\theta B} \left(\hat{l}_p(\tilde{k}(t)) \right)^{1-\beta} \left[1 - B \left(\hat{l}_p(\tilde{k}(t)) \right)^\beta \right] \right]^{\frac{1-\alpha}{\alpha}} \tilde{k}(t)$$

$$\hat{l}_p(\tilde{k}(t)) \stackrel{Def}{\Leftrightarrow} \frac{\left(\hat{l}_p(\tilde{k}(t)) \right)^{1-\beta}}{1 - \hat{l}_p(\tilde{k}(t))} = \frac{\theta B}{(1-\alpha)} \left[\tau + \frac{\psi}{\tilde{y}(\tilde{k}(t))} \right]$$

Equation (4.5) is the typical Euler equation and equation (4.6) shows that households have two sources of income: disposable (legal) income, $\tilde{y}(t)(1-\tau)$ and, rent seeking, $B(l_p)^\beta [\tau \tilde{y}(t) + \psi]$.

Lemma 4.1. *there is $\bar{B} \in \left(\frac{1-\beta}{1-\beta\alpha}, 1\right]$ such¹⁰ that $\forall B < \bar{B}$ the function $\hat{l}_p(\tilde{k}(t))$ exists and is strictly decreasing.*

We assume that $B < \bar{B}$, in order to guarantee the existence of the function $\hat{l}_p(\tilde{k}(t))$. When $\hat{l}_p(\cdot)$ exists, it is a strictly decreasing function¹¹. Thus, along the transition, when the capital in the *South* grows at a faster rate than the *North* (i.e., $\tilde{k}(t) = k^S(t)/y^N(t)$ increases), the amount of time devoted to rent-seeking activities decreases. The reason for this decline in rent-seeking is that when the *South* grows at a faster rate than in the *North*, the ratio international aid-wage goes down. This means that the reward for rent-seeking, which increases with international aid, grows slower than the reward for working, the wage. Consequently, agents have less incentive to devote time to rent-seeking activities and more incentive to devote time to work.

Figure 4.2 displays the Phase diagram associated with the above dynamic system. While the good BGP is always a saddle point and there is a unique equilibrium path converging to it, the bad BGP is either a focus or a source (see appendix 7.4). In the latter case, when the bad BGP is a source, there are multiple equilibria and some equilibrium paths converge to the bad equilibrium path. The dotted line shows the path that would converge to the trivial BGP characterized by zero capital and zero production. However, notice that consumption would be positive on the trivial BGP¹², since households would consume from international

¹⁰Note that $\frac{1-\beta}{1-\beta\alpha} > (1-\beta)$ since $\alpha < 1$. Thus, the subset of parameters which satisfy simultaneously this assumption and previous assumption, $B > (1-\beta)$, is non-empty.

¹¹In the case of $\bar{B} < B < 1$, there are multiple equilibria for low levels of capital. That is, for a given amount of capital there is more than one equilibrium. These “static” multiple equilibria arise due to the following mechanism: if a large amount of potential labor force is devoted to rent-seeking, income in the *South* is low and, consequently, the ratio aid-income is high; this promotes rent-seeking generating a vicious circle.

¹²In order for the trivial BGP to exist, the return on savings when \tilde{k} goes to zero should be lower than the discount rate of the utility:

$$\lim_{\tilde{k} \rightarrow 0} r(\tilde{k}) = \alpha A^{\frac{1}{\alpha}} \left[\frac{(1-\alpha)}{\theta B} (1-B) \right]^{\frac{1-\alpha}{\alpha}} - \delta \leq \rho$$

This condition means that households would contract debt if they could do it at the equilibrium interest rate, but they cannot since in our model there is no international capital market. Thus, households neither save nor incur debts. This means that the capital that they have is zero, and consequently production is zero and households would consume from the rent-seeking activities of international aid.

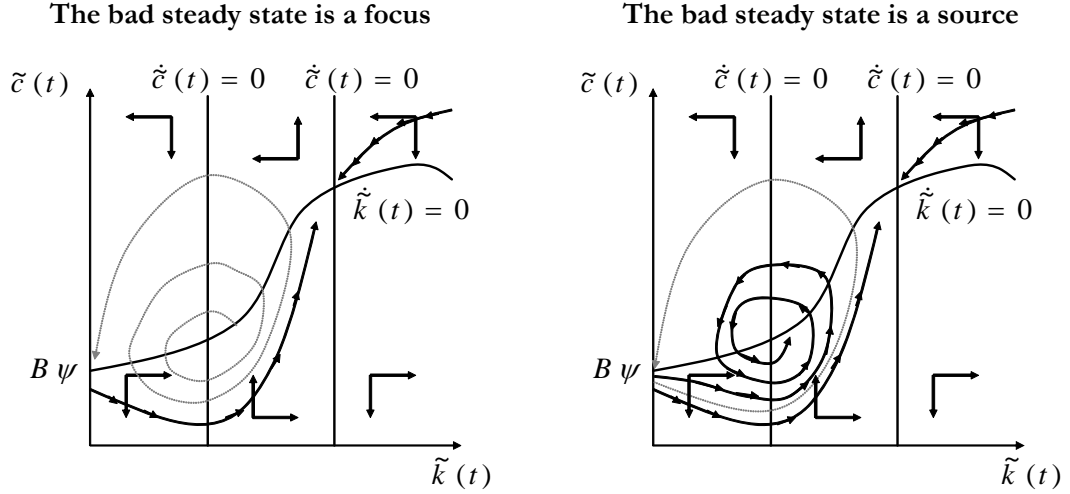


Figure 4.2: Transitional Dynamic to the BGP when aid is proportional to the income in the *North*

aid and would devote all their time to rent-seeking.

4.2.2. Bounded international aid

We now consider the ratio international aid-income in the *South* is bounded by the level that maximizes the growth rate of consumption in the *South*, a^* :

$$a(t) = \min \left\{ \frac{\psi}{\tilde{y}(t)}, a^* \right\} \Leftrightarrow aid(t) = \min \{ \psi y^N(t), a^* y^S(t) \}$$

The dynamic system which describes the behavior of the economy in the *South* in this case would be as follows:

$$\frac{\dot{\tilde{c}}(t)}{\tilde{c}(t)} = \frac{1}{\sigma} \left(\alpha \frac{\tilde{y}(\tilde{k}(t))}{\tilde{k}(t)} - \delta - \rho \right) - v^N \quad (4.7)$$

$$\begin{aligned} \dot{\tilde{k}}(t) = & \tilde{y}(k(t)) (1 - \tau) + B \left(\hat{l}_p(\tilde{k}(t)) \right)^\beta \left[\tau \tilde{y}(k(t)) + \min \left\{ \psi, a^* \tilde{y}(\tilde{k}(t)) \right\} \right] \\ & - (\delta + v^N) \tilde{k}(t) - \tilde{c}(t) \end{aligned} \quad (4.8)$$

where:

$$\begin{aligned}\tilde{y}(\tilde{k}(t)) &= A^{\frac{1}{\alpha}} \left[\frac{(1-\alpha)}{\theta B} (\hat{l}_p(\tilde{k}(t)))^{1-\beta} \left[1-B (\hat{l}_p(\tilde{k}(t)))^\beta \right] \right]^{\frac{1-\alpha}{\alpha}} \tilde{k}(t) \\ \hat{l}_p(\tilde{k}(t)) &\stackrel{Def}{\Leftrightarrow} \frac{(\hat{l}_p(\tilde{k}(t)))^{1-\beta}}{1-\hat{l}_p(\tilde{k}(t))} = \frac{\theta B}{(1-\alpha)} \left[\tau + \min \left\{ \frac{\psi}{\tilde{y}(t)}, a^* \right\} \right]\end{aligned}$$

The key difference between this dynamic system and the one described in subsection 4.2.1 in which international aid was unbounded, is that now, since the ratio government revenues-income in the *South* (T/y) is bounded above by $\tau + a^*$, the amount of time devoted to rent-seeking, which is an increasing function of T/y , is also bounded above by the threshold l_p^* defined as follows:

$$l_p^* \stackrel{Def}{\Leftrightarrow} \frac{(l_p^*)^{1-\beta}}{1-l_p^*} = \frac{\theta B}{(1-\alpha)} [\tau + a^*]$$

Since the amount of aid is bounded by a^* , the right part of the hump-shaped curve that relates growth in the *South* to aid disappears (see Figure 4.1). Thus, the relation between growth in the *South* and aid is strictly increasing in the interval $[0, a^*]$, equating the growth rate of the *North* only once. As a consequence, there is a unique BGP and a unique equilibrium path converging to it (see the appendix 7.5).

5. The case of *two-alike* countries

In previous sections, we investigated the effect of international aid provided by the *North* to the *South* assuming that the *North* has more potential growth. What would occur if the *South* and the *North* are identical in all aspects except in the initial level of capital? Is it possible that two identical countries may converge to two different long-run equilibria? To answer these questions, we will set up a world composed of two countries, the *North* and the *South*, which show identical economic features except for the initial amount of capital. Furthermore, we assume that the aid that each country receives is as follows:

$$\begin{aligned}aid^i(t) &= \psi (y^j(t) - y^i(t)); \quad i, j \in \{S, N\}; \quad i \neq j; \quad \psi > 0 \\ a^i(t) &= \frac{aid^i(t)}{y^i(t)} = \psi \left(\frac{y^j(t)}{y^i(t)} - 1 \right)\end{aligned}\tag{5.1}$$

where¹³ $\psi \in (0, \tau)$ and $aid^i(t)$ denotes the aid received by country i , and $a^i(t)$ denotes the ratio aid received by country i -income at country i . If the value of the aid is negative, this means that the country does not receive aid but it gives aid to the other country. That is, if the aid of country i is negative ($aid^i(t) < 0$), the country i is the donor and the other country, j , is the receptor country. Note that the sum of the aid of the two countries is always zero, since the amount that the receptor country receives is equal to the aid that the donor country gives:

$$\begin{aligned} aid^S(t) &= \psi (y^S(t) - y^N(t)) = -\psi (y^N(t) - y^S(t)) = -aid^N(t) \Rightarrow \\ aid^N(t) + aid^S(t) &= 0 \end{aligned}$$

The growth rate on the BGP would be as follows (see equation 3.1):

$$v^i = v\left(\frac{T^i}{y^i}\right) = v\left(\tau + \psi \left(\left(\frac{y^j}{y^i}\right) - 1\right)\right) \quad i, j \in \{S, N\}; i \neq j$$

where $v(\cdot)$ is defined as the function that relates the growth rate of consumption to the ratio public revenues-income:

$$v\left(\frac{T}{y}\right) = \frac{1}{\sigma} \left\{ \alpha A^{\frac{1}{\alpha}} \left[\frac{(1-\alpha)}{\theta B} \left(l_p \left(\frac{T}{y} \right) \right)^{1-\beta} \left[1 - B \left(l_p \left(\frac{T}{y} \right) \right)^\beta \right] \right]^{\frac{1-\alpha}{\alpha}} - \delta - \rho \right\}$$

where we have used equations (2.6) and (5.1). In order to have the same BGP in the *North* as in the *South*, growth rates of both countries should be the same. Thus, the following condition should hold:

$$\begin{aligned} v^S &= v^N \Leftrightarrow \\ v\left(\tau + \psi \left(\frac{1}{\tilde{y}} - 1\right)\right) &= v(\tau + \psi(\tilde{y} - 1)) \end{aligned} \quad (5.2)$$

where $\tilde{y}(t) = y^S(t)/y^N(t)$. It is obvious from equation (5.2) that there is always a BGP where $\tilde{y} = 1$. We will call such a BGP “symmetric” BGP, since the incomes of the *South* and the *North* equalize. Any other BGP where the income of the *South* and the *North* do not equalize, $\tilde{y} \neq 1$, will be called “asymmetric” BGP.

¹³The assumption that $\psi < \tau$ is needed so that the aid the donor country provides to the receptor country is always smaller than the revenues of the donor country: $aid^{receptor}(t) = \psi (y^{donor}(t) - y^{receptor}(t)) < \tau y^{donor}(t)$. This assumption is needed to satisfy budget constraint of the donor.

Note that if there is an asymmetric BGP in which for an instant the income in the *South* is lower than in the *North*, $\tilde{y}^{BGP1} < 1$, “BGP1”, then there is another asymmetric BGP in which the income in the South is higher than in the North where $\tilde{y}^{BGP2} = 1/\tilde{y}^{BGP1}$, “BGP2”. That is:

$$v\left(\tau + \psi\left(\frac{1}{\tilde{y}^{BGP1}} - 1\right)\right) = v\left(\tau + \psi\left(\tilde{y}^{BGP1} - 1\right)\right) \Leftrightarrow \quad (5.3)$$

$$v\left(\tau + \psi\left(\frac{1}{\tilde{y}^{BGP1}} - 1\right)\right) = v\left(\tau + \psi\left(\left(\frac{1}{\frac{1}{\tilde{y}^{BGP1}}}\right) - 1\right)\right) \Leftrightarrow \quad (5.4)$$

$$v\left(\tau + \psi\left(\tilde{y}^{BGP2} - 1\right)\right) = v\left(\tau + \psi\left(\left(\frac{1}{\tilde{y}^{BGP2}}\right) - 1\right)\right) \quad (5.5)$$

Thus, there is always an even number of asymmetric BGPs.

Proposition 5.1. *If one of the following sufficient conditions hold:*

- *Sufficient condition 1: $(1 - B) < (l_p(\tau - \psi))^{1-\beta} \left[1 - B(l_p(\tau - \psi))^\beta\right]$ and $\tau < a^*$*
 - *Sufficient condition 2: $(1 - B) > (l_p(\tau - \psi))^{1-\beta} \left[1 - B(l_p(\tau - \psi))^\beta\right]$ and $\tau > a^*$*
- then there are at least three BGPs, with only one of them being symmetric, $\tilde{y} = 1$. The growth rate along the symmetric BGP is higher than the growth rate in any of the asymmetric BGPs.*

We will focus on sufficient condition 1 for the discussion of the proposition. Note that if the ratio of income of the receptor country to income of the donor country ($y^{\text{receptor}}/y^{\text{donor}}$) is zero, the donor country would have the minimum possible disposable tax rate after aid is donated, equal to $\tau - \psi$ and, since l_p is increasing in the after aid tax rate (see equation 2.16), the donor country reaches the minimum possible amount of time devoted to predation, $l_p^{\text{donor}} = l_p(\tau - \psi)$. On the contrary, all units of labor in the receptor country would be devoted to predation¹⁴, $l_p^{\text{receptor}} = 1$. Sufficient condition 1 means that, in this case, in which

¹⁴ According to equation (2.16):

$$\lim_{\frac{x}{y} \rightarrow +\infty} \frac{l_p^{1-\beta}}{1 - l_p} = \lim_{\frac{x}{y} \rightarrow +\infty} \frac{\theta B}{(1 - \alpha)} \frac{T}{y} = +\infty \Rightarrow$$

$$\lim_{\frac{x}{y} \rightarrow +\infty} l_p = 1$$

the ratio $y^{\text{receptor}}/y^{\text{donor}}$ goes to zero and in which the receptor country devotes all units of labor to predation while the donor country devotes the minimum amount of labor to predation, the receptor country should grow more slowly than the donor country:

$$\lim_{y^{\text{receptor}}/y^{\text{donor}} \rightarrow 0} v^{\text{donor}} > \lim_{y^{\text{receptor}}/y^{\text{donor}} \rightarrow 0} v^{\text{receptor}} \Leftrightarrow \\ l_p (\tau - \psi)^{1-\beta} \left[1 - B (l_p (\tau - \psi))^{\beta} \right] > \lim_{l_p \rightarrow 1} l_p^{1-\beta} [1 - B l_p^{\beta}] = (1 - B)$$

The sufficient condition 1 also imposes that the tax rate should be smaller than the level of international aid that maximizes the growth rate of the *South*, a^* .

Figure 5.1 illustrates proposition 5.1. We plotted the curve that relates the growth rate of the *South* on the BGP with the convergence index \tilde{y} and, the curve that relates the growth rate of the *North* on the BGP with the convergence index \tilde{y} . On the BGP, the *South* and the *North* should grow at the same rate (see equation 5.2). Thus, when curves that represent the growth in the *North* and in the *South* cross, both countries grow at the same rate and consequently, cross points represent BGPs. The two curves cross three times: once at the symmetric BGP with convergence, $\tilde{y}^{\text{symmetric BGP}} = 1$ and, twice at two BGPs in which there is no convergence. In one of these asymmetric BGPs the *South* is poorer than the *North*, $\tilde{y}^{\text{asymmetric, BGP1}} < 1$ and in the other, the opposite occurs $\tilde{y}^{\text{asymmetric, BGP2}} > 1$. The two asymmetric BGPs imply the same growth rates and are symmetric between them in the following sense: $\tilde{y}^{\text{asymmetric, BGP1}} = 1/\tilde{y}^{\text{asymmetric, BGP2}}$ (see equations 5.3 to 5.5). Thus, we show that two identical countries may end up not converging to the same levels of per capita income simply because of the existence of international aid and the fact that they start with different initial levels of per capita capital. In this sense, we can state that history matters.

Another interesting feature to highlight is that growth rates along asymmetric BGPs (with no convergence and international aid) are lower than along the symmetric BGP (with convergence in per capita income and no international aid). The intuition of this result emerges clearly looking at Figure 5.1. In the asymmetric BGP, the receptor country (the *South* in the first asymmetric BGP and the *North* in the second one) receives an amount of international aid higher than a^* , $a^S > a^*$, which means that in the receptor country, the effect of international aid on promoting rent-seeking is stronger than the effect of providing more productive public good. Therefore, in this case international aid is harming growth in the receptor country. On the other hand, notice that the donor country (the

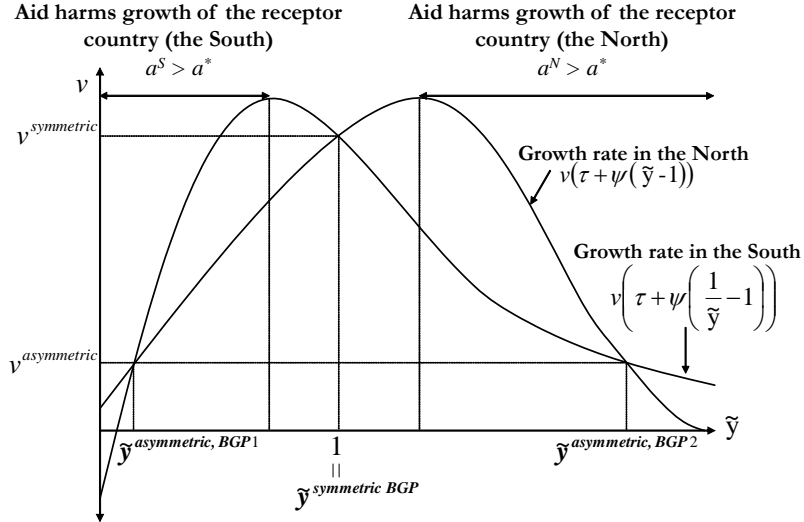


Figure 5.1: Relationship between growth and international aid in the *two-alike* countries case

North in the first asymmetric BGP and the *South* in the second) has a tax rate smaller than a^* and a ratio revenues-income lower than a^* , which means that an increase in the government revenue-income ratio in the donor country would increase the growth rate (in this case the effect of government revenues on the productive public good is stronger than the effect on rent-seeking). This means that international aid, which reduces the net revenues of the donor country, also harms the growth rate of the donor country. Summarizing, on the asymmetric BGP, international aid promotes rent-seeking in the receptor country and deviates resources from productive public investment to international aid in the donor country, reducing growth rates in both countries. Thus, international aid would harm the cross-country convergence in per capita income and the growth rate of the world economy.

6. Conclusions

Empirical evidence on the effect of foreign aid on economic growth is ambiguous and mixed. This paper analyzes the relationship between aid and growth in a context in which rent-seeking activities erode the effort of governments to provide a public good which generates growth. To do this, we build a model in which agents devote time to work and to rent-seeking activities to appropriate public revenues coming from non-distortionary taxes and foreign aid. The government uses the revenues after rent-seeking to finance a public good, which generates growth.

We show that the relationship between aid and growth is hump-shaped. When aid increases, three effects on growth appear: *(i)* it raises the government's resources and so the provision of the public good, increasing the productivity of the private sector and the growth rate, *(ii)* the increase in the government's resources raises the profitability of rent-seeking activities, reducing the amount of public revenues after rent-seeking and so reducing growth, *(iii)* since agents devote more time to rent-seeking activities, labor supply drops, thus, reducing also growth. We show that for low levels of aid the first effect prevails, while for high levels the other two effects are stronger. Moreover, we show that countries with bad institutions are the ones in which international aid is more likely to have a counterproductive effect on growth, since the encouraging effect that aid has on rent-seeking is stronger on them.

We examine the question of the convergence of the receptor country to the donor country. We analyze first that issue in an environment in which the receptor and the donor country are different. More precisely, the donor country has a larger potential long-run growth than the receptor country in the absence of international aid. Incorporating the assumption that international aid is a fraction of the income of the donor, we show that two BGPs emerge with different convergence level, where the degree of convergence is measured as the ratio between per capita income in the receptor country and per capita income in the donor country. We call the BGP with the higher convergence degree "good BGP" and the another one "bad BGP". We show that a drop in the amount of international aid may increase the per capita income convergence of the receptor country to the donor. More precisely, if the amount of aid per income of the receptor country is bounded above by the growth maximizing threshold, the receptor country will always converge to the good BGP, since the bad BGP disappears.

We then analyze the transition dynamic to the BGP. When the initial ratio

between per capita income of the receptor country and per capita income of the donor is smaller than the value on the BGP, the transitional dynamic is characterized by a decline in the time devoted to rent-seeking activities, since the reward for working, the wage, increases faster than the reward for rent-seeking activities, tax collection plus international aid received. Furthermore, the transitional dynamic may be characterized by multiple equilibria.

Finally, we examine the issue of convergence in a different environment in which the donor and the receptor country are totally identical except for their initial amount of capital. In this setting, the richer country donates international aid to the other one. We show that the existence of international aid may prevent the convergence of both countries to the same BGP, since international aid generates asymmetric BGPs, where there is no convergence in per capita income. In this sense, international aid from the richer to poorer country self-perpetuates. Furthermore, the growth rate in these asymmetric BGPs, without convergence in per capita income, is lower than the growth rate of the symmetric BGP, in which countries converge in per capita income. This last result is due to the fact that aid promotes rent-seeking in the receptor country and deviates resources from productive public investment to international aid in the donor country at the asymmetric BGP, harming growth in both countries. Thus, we show that under certain circumstances aid is bad for both growth and convergence.

7. Appendix

7.1. Growth rate and its maximum level:

Substituting the government budget constraint (equation 2.7) in the production function (equation 2.4) yields:

$$y = A \underbrace{\left[\left(\frac{T}{y} \right) [1 - Bl_p^\beta] y \right]^{1-\alpha}}_g k^\alpha (1 - l_p)^{1-\alpha} \Leftrightarrow$$

$$y = A^{\frac{1}{\alpha}} \left[\left(\frac{T}{y} \right) [1 - Bl_p^\beta] (1 - l_p) \right]^{\frac{1-\alpha}{\alpha}} k$$

If we now substitute equation (2.16) we obtain:

$$y = A^{\frac{1}{\alpha}} \left[\frac{(1 - \alpha)}{\theta B} l_p^{1-\beta} [1 - Bl_p^\beta] \right]^{\frac{1-\alpha}{\alpha}} k \quad (7.1)$$

Using equations (2.11) and (2.14) it yields:

$$v = \frac{1}{\sigma} \left(\alpha A^{\frac{1}{\alpha}} \left[\frac{(1-\alpha)}{\theta B} l_p^{1-\beta} [1 - Bl_p^\beta] \right]^{\frac{1-\alpha}{\alpha}} - \rho - \delta \right)$$

The maximum of this function is reached when:

$$\begin{aligned} \frac{\partial [l_p^{1-\beta} [1 - Bl_p^\beta]]}{\partial l_p} &= l_p^{1-\beta} [1 - Bl_p^\beta] \left[\frac{(1-\beta)}{l_p} - \frac{\beta Bl_p^{\beta-1}}{[1 - Bl_p^\beta]} \right] = 0 \Leftrightarrow (7.2) \\ (1-\beta) [1 - Bl_p^\beta] &= \beta Bl_p^\beta \Leftrightarrow l_p = \left(\frac{1-\beta}{B} \right)^{\frac{1}{\beta}} \Leftrightarrow \\ \left(\frac{T}{y} \right)^* &= \frac{(1-\alpha)}{\theta} \frac{(1-\beta)^{\frac{1-\beta}{\beta}}}{(B)^{\frac{1}{\beta}} - (1-\beta)^{\frac{1}{\beta}}} \end{aligned}$$

where in the last equation we use equation (2.16). It follows from the chain rule that:

$$\frac{\partial v}{\partial \left(\frac{T}{y} \right)} = \underbrace{\frac{\partial v}{\partial \left(l_p^{1-\beta} [1 - Bl_p^\beta] \right)}}_{\oplus} \frac{\partial [l_p^{1-\beta} [1 - Bl_p^\beta]]}{\partial l_p} \underbrace{\frac{\partial l_p}{\partial \left(\frac{T}{y} \right)}}_{\oplus}$$

Thus, the sign of the derivative of the growth rate, v , with respect to the ratio government revenues-income, T/y , is the same as the derivative defined in equation (7.2): it is positive for values smaller than $(T/y)^*$ and negative after reaching the maximum at $(T/y)^*$. Thus, there is a hump-shaped relationship between the growth rate and the ratio revenues-income.

The relationship between the growth rate and the index of institutional weakness would be given by the following derivative:

$$\begin{aligned} \frac{\partial \left[\frac{l_p^{1-\beta} [1 - Bl_p^\beta]}{B} \right]}{\partial B} &= - \frac{l_p^{1-\beta} [1 - Bl_p^\beta]}{B} \times \\ &\left\{ \frac{l_p^\beta}{[1 - Bl_p^\beta]} + \frac{1}{B} + \left[\frac{l_p^\beta}{[1 - Bl_p^\beta]} - (1-\beta) \frac{1}{l_p} \right] \frac{\partial l_p \left(\frac{T}{y}, B, (1-\alpha) \right)}{\partial B} \right\} \end{aligned}$$

Using equations (2.17) and (2.19) it follows that:

$$\begin{aligned}
\frac{\partial l_p \left(\frac{T}{y}, B, (1-\alpha) \right)}{\partial B} &= \frac{1}{B} \frac{l_p^{1-\beta}}{1-l_p} \frac{[1-l_p]^2 l_p^\beta}{(1-\beta)[1-l_p] + l_p} = \\
&= \frac{1}{B} \frac{[1-l_p] l_p}{(1-\beta)[1-l_p] + l_p} \Rightarrow \\
(1-\beta) \frac{1}{l_p} \frac{\partial l_p \left(\frac{T}{y}, B, (1-\alpha) \right)}{\partial B} &= \frac{1}{B} \frac{(1-\beta)[1-l_p]}{(1-\beta)[1-l_p] + l_p} < \frac{1}{B} \Rightarrow \\
\frac{\partial \left[\frac{l_p^{1-\beta} [1-B l_p^\beta]}{B} \right]}{\partial B} &< - \frac{l_p^{1-\beta} [1-B l_p^\beta]}{B} \times \\
\left\{ \frac{l_p^\beta}{[1-B l_p^\beta]} + \frac{1}{B} + \frac{l_p^\beta}{[1-B l_p^\beta]} \frac{\partial l_p \left(\frac{T}{y}, B, (1-\alpha) \right)}{\partial B} - \frac{1}{B} \right\} &< 0
\end{aligned}$$

It follows from the previous equation that:

$$\frac{\partial \left[\frac{l_p^{1-\beta} [1-B l_p^\beta]}{B} \right]}{\partial B} < 0$$

which implies that:

$$\frac{\partial v}{\partial B} = \underbrace{\frac{\partial v}{\partial \left(l_p^{1-\beta} [1-B l_p^\beta] \right)}}_{\oplus} \underbrace{\frac{\partial [l_p^{1-\beta} [1-B l_p^\beta]]}{\partial B}}_{\ominus} < 0$$

7.2. Transitional dynamics: International aid proportional to the income of the *North*

The Euler Equation for the *South* is as follows (see equation 2.14):

$$\frac{\dot{c}^S(t)}{c^S(t)} = \frac{1}{\sigma} (r^S(t) - \rho)$$

Since we have defined $\tilde{c}(t) = c^S(t)/y^N(t)$ and so $\frac{\dot{\tilde{c}}(t)}{\tilde{c}(t)} = \frac{c^S(t)}{c^S(t)} - v^N$ and using equation (2.11) we obtain the first equation:

$$\frac{\dot{\tilde{c}}(t)}{\tilde{c}(t)} = \frac{1}{\sigma} \left(\alpha \frac{\tilde{y}(\tilde{k}(t))}{\tilde{k}(t)} - \delta - \rho \right) - v^N$$

Similarly, from the budget constraint of the household's optimization problem (equation 2.12) at *South* and considering equation (2.5), equation (4.4), definition $\tilde{k}(t) = k^S(t)/y^N(t)$, the clearing condition in the capital market, $k(t) = b(t)$, and the fact that the time devoted to rent-seeking activity, l_p^i , is the same for all agents in the *South*, $l_p^i(t) = l_p^j(t) = l_p(t)$ we obtain:

$$\dot{\tilde{k}}(t) = r^S(t)\tilde{k}(t) + w^S(t)\frac{(1-l_p(t))}{y^N(t)} + Bl_p(t)^\beta [\tau\tilde{y}(k(t)) + \psi] - v^N\tilde{k}(t) - \tau\tilde{y}(t) - \tilde{c}(t)$$

Then, using equations (2.10), (2.11) and (7.3) we obtain:

$$\dot{\tilde{k}}(t) = (1-\tau)\tilde{y}(t) + Bl_p(t)^\beta [\tau\tilde{y}(k(t)) + \psi] - (\delta + v^N)\tilde{k}(t) - \tilde{c}(t)$$

and, from equations (7.1), (4.4) and (2.6) we obtain:

$$\begin{aligned} \frac{y^S(t)}{y^N(t)} &= A^{\frac{1}{\alpha}} \left[\frac{(1-\alpha)}{\theta B} (l_p(t))^{1-\beta} \left[1 - B(l_p(t))^\beta \right] \right]^{\frac{1-\alpha}{\alpha}} \frac{k(t)}{y^N(t)} \Rightarrow \\ \tilde{y}(t) &= A^{\frac{1}{\alpha}} \left[\frac{(1-\alpha)}{\theta B} (l_p(t))^{1-\beta} \left[1 - B(l_p(t))^\beta \right] \right]^{\frac{1-\alpha}{\alpha}} \tilde{k}(t) \end{aligned} \quad (7.3)$$

Finally, from equations (2.6), (2.16), (4.4) (7.3), we obtain:

$$\frac{(l_p(t))^{1-\beta}}{1-l_p(t)} = \frac{\theta B}{(1-\alpha)} \left(\tau + \frac{\psi}{A^{\frac{1}{\alpha}} \left[\frac{(1-\alpha)}{\theta B} (l_p(t))^{1-\beta} \left[1 - B(l_p(t))^\beta \right] \right]^{\frac{1-\alpha}{\alpha}} \tilde{k}(t)} \right)$$

Thus, the amount of time devoted to work in the market is a function of the ratio capital in the *South* to income in the *North* $\tilde{k}(t)$:

$$l_p(t) = \hat{l}_p(\tilde{k}(t))$$

7.3. Proof of Lemma 4.1

From, appendix 7.2 we know:

$$\begin{aligned} \widehat{l}_p(\widetilde{k}) &\stackrel{Def}{\Leftrightarrow} \\ \frac{\left(\widehat{l}_p(\widetilde{k})\right)^{1-\beta}}{1-\widehat{l}_p(\widetilde{k})} &= \frac{\theta B}{(1-\alpha)} \left(\tau + \frac{\psi}{A^{\frac{1}{\alpha}} \left[\frac{(1-\alpha)}{\theta B} \left(\widehat{l}_p(\widetilde{k})\right)^{1-\beta} \left[1-B\left(\widehat{l}_p(\widetilde{k})\right)^\beta\right]\right]^{\frac{1-\alpha}{\alpha}} \widetilde{k}} \right) \Leftrightarrow \\ \widetilde{k} = f(l_p) &= \frac{\frac{\theta B}{(1-\alpha)} \psi}{A^{\frac{1}{\alpha}} \left(\frac{(l_p)^{1-\beta}}{1-l_p} - \frac{\theta B}{(1-\alpha)} \tau \right) \left[\frac{(1-\alpha)}{\theta B} (l_p)^{1-\beta} \left[1-B(l_p)^\beta\right]\right]^{\frac{1-\alpha}{\alpha}}} \end{aligned}$$

If the above function $f(l_p)$ is invertible, then its inverse is $\widehat{l}_p(\widetilde{k}) = f^{-1}(\widetilde{k})$. Thus, to prove the existence of $\widehat{l}_p(\widetilde{k})$, we will prove that $f(l_p)$ is invertible. We concentrate on the relevant range in which \widetilde{k} is positive, that is when $\frac{(l_p)^{1-\beta}}{1-l_p} - \frac{\theta B}{(1-\alpha)} \tau > 0 \Leftrightarrow l_p > l_p^{\min}$, where l_p^{\min} is defined as follows:

$$l_p^{\min} \stackrel{Def}{\Leftrightarrow} \frac{(l_p)^{1-\beta}}{1-l_p} = \frac{\theta B}{(1-\alpha)} \tau \quad (7.4)$$

The derivative of $f(l_p)$ is as follows

$$\begin{aligned} \frac{\partial f(l_p)}{\partial l_p} &= \quad (7.5) \\ -\frac{f(l_p)}{l_p} &\left[\frac{\frac{(l_p)^{1-\beta}}{1-l_p}}{\frac{(l_p)^{1-\beta}}{1-l_p} - \frac{\theta B}{(1-\alpha)} \tau} \left[(1-\beta) + \frac{l_p}{1-l_p} \right] + \frac{1-\alpha}{\alpha} \left[(1-\beta) - \frac{\beta B (l_p)^\beta}{1-B(l_p)^\beta} \right] \right] < \\ -\frac{f(l_p)}{l_p} &\frac{1}{\alpha} \left[\alpha \frac{l_p}{1-l_p} + (1-\beta) - \frac{\beta (1-\alpha) B (l_p)^\beta}{1-B(l_p)^\beta} \right] \end{aligned}$$

where in the inequality we used the fact that $\frac{\frac{(l_p)^{1-\beta}}{1-l_p}}{\frac{(l_p)^{1-\beta}}{1-l_p} - \frac{\theta B}{(1-\alpha)} \tau} > 1$.

The rest of the proof is based on three remarks: Remark 1 proves that the function $f(l_p)$ is invertible and continuous, Remark 2 proves that the function $f(l_p)$ is strictly decreasing, and Remark 3 proves that $\overline{B} < 1$ when $\frac{1-\alpha}{\alpha} > 1$ (see footnote 4).

Remark 1. $x - \frac{\beta(1-\alpha)B(l_p)^\beta}{1-B(l_p)^\beta} \geq 0 \Leftrightarrow l_p \leq \left(\frac{1}{B} \frac{x}{x+\beta(1-\alpha)}\right)^{\frac{1}{\beta}}$

Thus, if $l_p \leq \left(\frac{1}{B} \frac{1-\beta}{1-\beta+\beta(1-\alpha)}\right)^{\frac{1}{\beta}} = \left(\frac{1}{B} \frac{1-\beta}{1-\beta\alpha}\right)^{\frac{1}{\beta}}$ then $(1-\beta) - \frac{\beta(1-\alpha)B(l_p)^\beta}{1-B(l_p)^\beta} > 0 \Rightarrow \frac{\partial f(l_p)}{\partial l_p} < 0$ (see 7.5).

When $\left(\frac{1}{B} \frac{1-\beta}{1-\beta\alpha}\right)^{\frac{1}{\beta}} > 1 \Leftrightarrow B < \frac{1-\beta}{1-\beta\alpha}$ then $\forall l_p \in [l_p^{\min}, 1] : \frac{\partial f(l_p)}{\partial l_p} < 0$.

Let's define the following function for the case in which $B \geq \frac{1-\beta}{1-\beta\alpha}$:

$$\begin{aligned} \phi(B) = & \max_{l_p \in \left[\left(\frac{1}{B} \frac{1-\beta}{1-\beta\alpha}\right)^{\frac{1}{\beta}}, 1\right]} \frac{l_p \left(1 - B(l_p)^\beta\right) (1 - l_p) \frac{\partial f(l_p)}{\partial l_p}}{f(l_p)} = \\ & \max_{l_p \in \left[\left(\frac{1}{B} \frac{1-\beta}{1-\beta\alpha}\right)^{\frac{1}{\beta}}, 1\right]} - \left[\frac{(l_p)^{1-\beta} \left[(1-\beta) \left(1 - B(l_p)^\beta\right) (1 - l_p) + l_p \left(1 - B(l_p)^\beta\right) \right]}{(l_p)^{1-\beta} - \frac{\theta B}{(1-\alpha)} \tau (1 - l_p)} + \right. \\ & \left. \frac{1-\alpha}{\alpha} \left[(1-\beta) \left(1 - B(l_p)^\beta\right) (1 - l_p) - \beta B(l_p)^\beta (1 - l_p) \right] \right] \end{aligned}$$

Note that the objective function is a continuous function for $l_p > l_p^{\min}$, since the objective function is continuous in the range in which it is maximized. Furthermore, since $B \geq \frac{1-\beta}{1-\beta\alpha}$ the correspondence $l_p \in \left[\left(\frac{1}{B} \frac{1-\beta}{1-\beta\alpha}\right)^{\frac{1}{\beta}}, 1\right]$ is also continuous. Thus, the Maximum Theorem implies that $\phi(B)$ is a continuous function. Furthermore:

$$\begin{aligned} \phi\left(\frac{1-\beta}{1-\beta\alpha}\right) = & \max_{l_p \in [1,1]} - \left[\frac{(l_p)^{1-\beta} \left[(1-\beta) \left(1 - \left(\frac{1-\beta}{1-\beta\alpha}\right) (l_p)^\beta\right) (1-l_p) + l_p \left(1 - \left(\frac{1-\beta}{1-\beta\alpha}\right) (l_p)^\beta\right) \right]}{(l_p)^{1-\beta} - \frac{\theta B}{(1-\alpha)} \tau (1 - l_p)} + \right. \\ & \left. \frac{1-\alpha}{\alpha} \left[(1-\beta) \left(1 - \left(\frac{1-\beta}{1-\beta\alpha}\right) (l_p)^\beta\right) (1-l_p) - \beta \left(\frac{1-\beta}{1-\beta\alpha}\right) (l_p)^\beta (1-l_p) \right] \right] = \\ & - \left(1 - \frac{1-\beta}{1-\beta\alpha}\right) < 0 \end{aligned}$$

Thus, it follows from the continuity of $\phi(B)$ that either $\forall B \leq 1 : \phi(B) < 0$ or $\exists \bar{B} \in \left(\frac{1-\beta}{1-\beta\alpha}, 1\right]$ such that $\forall B < \bar{B} : \phi(B) < 0$ and if $B = \bar{B} : \phi(B) = 0$.

Remark 2. When $B < \overline{B}$ the function $f(l_p)$ is strictly decreasing, which also means that the inverse of such function, $\widehat{l}_p(\widetilde{k}) = f^{-1}(\widetilde{k})$, is also strictly decreasing.

Remark 3. If $\frac{1-\alpha}{\alpha} > 1$ then $\overline{B} < 1$.

To prove this, let's define the function $g(l_p)$ as follows:

$$g(l_p, B) = \frac{(1 - B (l_p)^\beta) l_p \partial f(l_p, B)}{f(l_p, B) \partial l_p} =$$

$$- \left[\frac{(l_p)^{1-\beta} \left[(1 - \beta) (1 - B (l_p)^\beta) + \frac{l_p (1 - B (l_p)^\beta)}{(1 - l_p)} \right]}{(l_p)^{1-\beta} - \frac{\theta B}{(1-\alpha)} \tau (1 - l_p)} \right] +$$

$$\frac{1 - \alpha}{\alpha} \left[(1 - \beta) (1 - B (l_p)^\beta) - \beta B (l_p)^\beta \right]$$

Note that

$$\lim_{B \rightarrow 1} g(B^\gamma, B) =$$

$$- \lim_{B \rightarrow 1} \left[\frac{B^{\gamma(1-\beta)} \left[(1 - \beta) (1 - B^{1+\gamma\beta}) + \frac{B^\gamma (1 - B^{1+\gamma\beta})}{(1 - B^\gamma)} \right]}{B^{\gamma(1-\beta)} - \frac{\theta B}{(1-\alpha)} \tau (1 - B^\gamma)} \right] +$$

$$\frac{1 - \alpha}{\alpha} \left[(1 - \beta) (1 - B^{1+\gamma\beta}) - \beta B^{1+\gamma\beta} \right] =$$

$$- \left[\frac{1 + \gamma\beta}{\gamma\beta} - \frac{1 - \alpha}{\alpha} \right] \beta$$

where $\gamma \in \mathfrak{R}_{++}$. Since $\lim_{\gamma \rightarrow +\infty} \frac{1+\gamma\beta}{\gamma\beta} = 1$, if $\frac{1-\alpha}{\alpha} > 1$ then a large enough $\widetilde{\gamma}$ always exists such that:

$$\lim_{B \rightarrow 1} g(B^{\widetilde{\gamma}}, B) = - \left[\frac{1 + \widetilde{\gamma}\beta}{\widetilde{\gamma}\beta} - \frac{1 - \alpha}{\alpha} \right] \beta > 0$$

Thus, there exist $\widetilde{B} < 1$ such that $g(\widetilde{B}^{\widetilde{\gamma}}, \widetilde{B}) > 0$. It follows from the definition of $g(l_p, B)$ that:

$$\frac{\partial f(\widetilde{B}^{\widetilde{\gamma}}, \widetilde{B})}{\partial l_p} > 0$$

Thus: $\bar{B} < \tilde{B} < 1$.

7.4. Linearized dynamic system

If we linearize the dynamic system (4.5)-(4.6) around the steady state, we get:

$$\begin{bmatrix} \dot{\tilde{c}}(t) \\ \dot{\tilde{k}}(t) \end{bmatrix} = \begin{bmatrix} 0 & \frac{\partial v_{\tilde{c}}(\tilde{k}^{BGP})}{\partial \tilde{k}} \tilde{c}^{BGP} \\ -1 & a_{22}^{BGP} \end{bmatrix} \begin{bmatrix} \tilde{c}(t) - \tilde{c}^{BGP} \\ \tilde{k}(t) - \tilde{k}^{BGP} \end{bmatrix}$$

where $v_{\tilde{c}}$ is the growth rate of \tilde{c} :

$$v_{\tilde{c}}(\tilde{k}) = \frac{1}{\sigma} \left(r(\tilde{k}) - \rho \right) - v^N$$

a_{22}^{BGP} is defined as follows:

$$\begin{aligned} a_{22}^{BGP} &= \frac{1}{\alpha} \left[r(\tilde{k}^{BGP}) + \frac{\partial r(\tilde{k}^{BGP})}{\partial \tilde{k}} \tilde{k}^{BGP} \right] \left(1 - \tau(1 - B(l_p^{BGP})^\beta) \right) + \\ & \left[\tau \tilde{y}(\tilde{k}(t)) + \psi \right] \beta B (\hat{l}_p^{BGP})^{\beta-1} \frac{\partial \hat{l}_p(\tilde{k}^{BGP})}{\partial \tilde{k}} - (\delta + v^N) = \\ & \frac{1}{\alpha} \left[\sigma v^N + \rho + \frac{\partial r(\tilde{k}^{BGP})}{\partial \tilde{k}} \tilde{k}^{BGP} \right] \left(1 - \tau(1 - B(l_p^{BGP})^\beta) \right) + \\ & \left[\tau \tilde{y}(\tilde{k}(t)) + \psi \right] \beta B (\hat{l}_p^{BGP})^{\beta-1} \frac{\partial \hat{l}_p(\tilde{k}^{BGP})}{\partial \tilde{k}} - (\delta + v^N) \end{aligned}$$

The characteristic equation associated with the above linear dynamic system is as follows:

$$\begin{aligned} & \begin{vmatrix} -\lambda & \frac{\partial v_{\tilde{c}}(\tilde{k}^{BGP})}{\partial \tilde{k}} \tilde{c}^{BGP} \\ -1 & a_{22}^{BGP} - \lambda \end{vmatrix} = 0 \\ & \lambda^2 + a_{22}^{BGP} \lambda + \frac{\partial v_{\tilde{c}}(\tilde{k}^{BGP})}{\partial \tilde{k}} \tilde{c}^{BGP} = 0 \end{aligned}$$

The roots associated with the above linear dynamic system are as follows:

$$\lambda = \frac{-a_{22}^{BGP} \pm \sqrt{(a_{22}^{BGP})^2 - 4 \frac{\partial v_{\tilde{c}}(\tilde{k}^{BGP})}{\partial \tilde{k}} \tilde{c}^{BGP}}}{2} \quad (7.6)$$

It follows from lemma 4.1, equation (7.4) and assumptions (4.1), (4.2) and (4.3) that:

$$\begin{aligned}
\lim_{\tilde{k} \rightarrow +\infty} \tilde{y}(\tilde{k}) &= \lim_{\tilde{k} \rightarrow +\infty} A^{\frac{1}{\alpha}} \left[\frac{(1-\alpha)}{\theta B} (\widehat{l}_p(\tilde{k}))^{1-\beta} \left[1-B (\widehat{l}_p(\tilde{k}))^\beta \right] \right]^{\frac{1-\alpha}{\alpha}} \tilde{k} = \\
\lim_{\tilde{k} \rightarrow +\infty} A^{\frac{1}{\alpha}} \left[\frac{(1-\alpha)}{\theta B} (l_p^{\min})^{1-\beta} \left[1-B (l_p^{\min})^\beta \right] \right]^{\frac{1-\alpha}{\alpha}} \tilde{k} &= +\infty \Rightarrow \\
\lim_{\tilde{k} \rightarrow +\infty} \left[\tau + \frac{\psi}{\tilde{y}(\tilde{k})} \right] = \tau &\Rightarrow \lim_{\tilde{k} \rightarrow +\infty} v_{\tilde{c}}(\tilde{k}) = v_{\text{no aid}}^S - v^N < 0 \quad (7.7)
\end{aligned}$$

Moreover, $\lim_{\tilde{k} \rightarrow 0} \tilde{y}(\tilde{k}) = 0$.

Remark, $\widehat{l}_p(\tilde{k}) \stackrel{Def}{\Leftrightarrow} \frac{(\widehat{l}_p(\tilde{k}))^{1-\beta}}{1-\widehat{l}_p(\tilde{k})} = \frac{\theta B}{(1-\alpha)} \left[\tau + \frac{\psi}{\tilde{y}(\tilde{k})} \right]$. Thus,

$$\begin{aligned}
\lim_{\tilde{k} \rightarrow 0} \tilde{y}(\tilde{k}) = 0 &\Rightarrow \lim_{\tilde{k} \rightarrow 0} \frac{\theta B}{(1-\alpha)} \left[\tau + \frac{\psi}{\tilde{y}(\tilde{k})} \right] = +\infty \Rightarrow \\
&\Rightarrow \lim_{\tilde{k} \rightarrow 0} \widehat{l}_p(\tilde{k}) = 1 \Rightarrow \\
&\Rightarrow \lim_{\tilde{k} \rightarrow 0} r(\tilde{k}) = \alpha A^{\frac{1}{\alpha}} \left[\frac{(1-\alpha)}{\theta B} [1-B] \right]^{\frac{1-\alpha}{\alpha}} - \delta \Rightarrow \\
&\Rightarrow \lim_{\tilde{k} \rightarrow 0} v_{\tilde{c}}(\tilde{k}) = \frac{1}{\sigma} \left(\alpha A^{\frac{1}{\alpha}} \left[\frac{(1-\alpha)}{\theta B} [1-B] \right]^{\frac{1-\alpha}{\alpha}} - \delta - \rho \right) - v^N < 0 \quad (7.8) \\
\tilde{k}^* \stackrel{def}{\Leftrightarrow} \frac{\psi}{\tilde{y}(\tilde{k}^*)} = a^* &\Rightarrow v_{\tilde{c}}(\tilde{k}^*) = \max_{\frac{T}{Y}} v^S - v^N > 0 \quad (7.9)
\end{aligned}$$

Thus: (i) when \tilde{k} is small enough $v_{\tilde{c}}(\tilde{k}) < 0$; (ii) when \tilde{k} is large enough $v_{\tilde{c}}(\tilde{k}) < 0$; (iii) There are values of \tilde{k} (for instance, \tilde{k}^*) in which $v_{\tilde{c}}(\tilde{k}) > 0$; (iv) we know that there are two BGPs, consequently $v_{\tilde{c}}(\tilde{k}) = 0$ for two values \tilde{k}^{bad} and \tilde{k}^{good} , where $\tilde{k}^{bad} < \tilde{k}^{good}$. Thus, when \tilde{k} increases $v_{\tilde{c}}(\tilde{k})$ is negative for small values

of \tilde{k} (\tilde{k} smaller than \tilde{k}^{bad}), then it becomes zero when $\tilde{k} = \tilde{k}^{bad}$, then it becomes positive, then zero again when $\tilde{k} = \tilde{k}^{good}$, and finally, negative again for levels of \tilde{k} higher than \tilde{k}^{good} . Thus, generically:

$$\frac{\partial v_{\tilde{c}}(\tilde{k}^{bad})}{\partial \tilde{k}} > 0$$

$$\frac{\partial v_{\tilde{c}}(\tilde{k}^{good})}{\partial \tilde{k}} < 0$$

Let's consider first the good BGP. We now know that $\sqrt{(a_{22}^{good})^2 - 4 \frac{\partial v_{\tilde{c}}(\tilde{k}^{good})}{\partial \tilde{k}} \tilde{c}^{good}} > |a_{22}^{good}| > 0$. Thus, according to equation (7.6) the roots of the linearized linear system in the surrounding of the good BGP are real ones, one of them being negative and the other positive, which means that the good BGP is a saddle point.

Let's turn now to the bad BGP. We have four possible cases:

1. $(a_{22}^{bad})^2 - 4 \frac{\partial v_{\tilde{c}}(\tilde{k}^{bad})}{\partial \tilde{k}} \tilde{c}^{bad} > 0$ and $a_{22}^{bad} > 0$, both roots are real and negative, then the bad BGP is a source.
2. $(a_{22}^{bad})^2 - 4 \frac{\partial v_{\tilde{c}}(\tilde{k}^{bad})}{\partial \tilde{k}} \tilde{c}^{bad} > 0$ and $a_{22}^{bad} < 0$, both roots are real and positive, then the bad BGP is a focus.
3. $(a_{22}^{bad})^2 - 4 \frac{\partial v_{\tilde{c}}(\tilde{k}^{bad})}{\partial \tilde{k}} \tilde{c}^{bad} < 0$ and $a_{22}^{bad} > 0$, both roots are complex, and the real part of the root $-a_{22}^{bad}$ is negative, then the bad BGP is a source.
4. $(a_{22}^{bad})^2 - 4 \frac{\partial v_{\tilde{c}}(\tilde{k}^{bad})}{\partial \tilde{k}} \tilde{c}^{bad} < 0$ and $a_{22}^{bad} < 0$, both roots are complex, and the real part of the root $-a_{22}^{bad}$ is positive, then the bad BGP is a focus.

7.5. Transitional dynamics: Bounded international aid

We have defined l_p^* as the amount of time devoted to rent-seeking which maximizes the growth rate of consumption:

$$\begin{aligned}
l_p^* &= \arg \max \frac{1}{\sigma} \left(\alpha A^{\frac{1}{\alpha}} \left[\frac{(1-\alpha)}{\theta B} (l_p)^{1-\beta} \left[1-B(l_p)^\beta \right] \right]^{\frac{1-\alpha}{\alpha}} - \delta - \rho \right) \Rightarrow \\
\frac{\partial v_c}{\partial l_p} &= \frac{\partial \left[\frac{1}{\sigma} \left(\alpha A^{\frac{1}{\alpha}} \left[\frac{(1-\alpha)}{\theta B} (l_p^*)^{1-\beta} \left[1-B(l_p^*)^\beta \right] \right]^{\frac{1-\alpha}{\alpha}} - \delta - \rho \right) \right]}{\partial l_p} = \\
\frac{1}{\sigma} \alpha A^{\frac{1}{\alpha}} \left[\frac{(1-\alpha)}{\theta B} (l_p^*)^{1-\beta} \left[1-B(l_p^*)^\beta \right] \right]^{\frac{1-\alpha}{\alpha}} \frac{1-\alpha}{\alpha} \frac{1}{l_p^*} \left[(1-\beta) - \frac{\beta B (l_p^*)^\beta}{1-B(l_p^*)^\beta} \right] &= 0
\end{aligned}$$

Since the function $\frac{\beta B (l_p)^\beta}{1-B(l_p)^\beta}$ is increasing, it follows that :

$$\begin{aligned}
\forall l_p < l_p^* \quad \left[(1-\beta) - \frac{\beta B (l_p)^\beta}{1-B(l_p)^\beta} \right] &> \left[(1-\beta) - \frac{\beta B (l_p^*)^\beta}{1-B(l_p^*)^\beta} \right] = 0 \Rightarrow \\
\frac{\partial v_c}{\partial l_p} &= \frac{\partial \left[\frac{1}{\sigma} \left(\alpha A^{\frac{1}{\alpha}} \left[\frac{(1-\alpha)}{\theta B} (l_p)^{1-\beta} \left[1-B(l_p)^\beta \right] \right]^{\frac{1-\alpha}{\alpha}} - \delta - \rho \right) \right]}{\partial l_p} = \\
\frac{1}{\sigma} \alpha A^{\frac{1}{\alpha}} \left[\frac{(1-\alpha)}{\theta B} (l_p)^{1-\beta} \left[1-B(l_p)^\beta \right] \right]^{\frac{1-\alpha}{\alpha}} \frac{1-\alpha}{\alpha} \frac{1}{l_p} \left[(1-\beta) - \frac{\beta B (l_p)^\beta}{1-B(l_p)^\beta} \right] &> 0 \quad (7.10)
\end{aligned}$$

In this case the amount of labor devoted to predation is bounded above:

$$l_p(t) = \min \left\{ \widehat{l}_p(\widetilde{k}), l_p^* \right\}, \text{ where: } l_p^* \stackrel{Def}{\Leftrightarrow} \frac{(l_p^*)^{1-\beta}}{1-l_p^*} = \frac{\theta B}{(1-\alpha)} [\tau + a^*]$$

Let's define \widetilde{k}^* as follows:

$$\widetilde{k}^* \stackrel{def}{\Leftrightarrow} \frac{\psi}{\widetilde{y}(\widetilde{k}^*)} = a^* \Rightarrow v_c(\widetilde{k}^*) = \max_{\frac{\tau}{y}} v^S - v^N > 0$$

If $\widetilde{k} \leq \widetilde{k}^*$:

$$l_p(t) = \min \left\{ \widehat{l}_p(\widetilde{k}), l_p^* \right\} = l_p^* \Rightarrow v_c(\widetilde{k}) = v_c(\widetilde{k}^*) = \max_{\frac{\tau}{y}} v^S - v^N > 0$$

If $\tilde{k} > \tilde{k}^*$:

$$l_p(t) = \min \left\{ \hat{l}_p(\tilde{k}), l_p^* \right\} = \hat{l}_p(\tilde{k}) < l_p^*$$

Thus, when $\tilde{k} > \tilde{k}^*$, we know that $l_p < l_p^*$. Then it follows from equation (7.10) that

$$\begin{aligned} \frac{\partial v_c}{\partial l_p} &= \frac{\partial \left[\frac{1}{\sigma} \left(\alpha A^{\frac{1}{\alpha}} \left[\frac{(1-\alpha)}{\theta B} (l_p)^{1-\beta} \left[1 - B(l_p)^\beta \right] \right]^{\frac{1-\alpha}{\alpha}} - \delta - \rho \right) \right]}{\partial l_p} > 0 \Rightarrow \\ &\frac{\partial \left[\alpha A^{\frac{1}{\alpha}} \left[\frac{(1-\alpha)}{\theta B} (l_p)^{1-\beta} \left[1 - B(l_p)^\beta \right] \right]^{\frac{1-\alpha}{\alpha}} \right]}{\partial l_p} > 0 \Rightarrow \\ \frac{\partial r(\tilde{k})}{\partial \tilde{k}} &= \frac{\partial \left[\alpha A^{\frac{1}{\alpha}} \left[\frac{(1-\alpha)}{\theta B} \left(\hat{l}_p(\tilde{k}(t)) \right)^{1-\beta} \left[1 - B \left(\hat{l}_p(\tilde{k}(t)) \right)^\beta \right] \right]^{\frac{1-\alpha}{\alpha}} \right]}{\partial l_p} \frac{\partial \hat{l}_p(\tilde{k}(t))}{\partial \tilde{k}} < 0 \end{aligned}$$

This means that the interest rate is constant for $\tilde{k} \leq \tilde{k}^*$ (with the growth rate of \tilde{c} being positive) and strictly decreasing for $\tilde{k} > \tilde{k}^*$. This means that a unique BGP exists such that $\tilde{k}^{BGP} > \tilde{k}^*$. Since $\tilde{k}^{BGP} > \tilde{k}^*$, then $\frac{\partial r(\tilde{k}^{BGP})}{\partial \tilde{k}} < 0$. As it was shown for the case in which international aid is proportional to the income of the *North*, when $\frac{\partial r(\tilde{k}^{BGP})}{\partial \tilde{k}} < 0$ the BGP is a saddle point.

7.6. Proof of Proposition 5.1

7.6.1. Sufficient condition 1: $(1 - B) < (l_p(\tau - \psi))^{1-\beta} \left[1 - B(l_p(\tau - \psi))^\beta \right]$
and $\tau < a^*$

The following condition should hold on the BGP (see equation 5.2):

$$F(\tilde{y}) = v \left(\tau + \psi \left(\frac{1}{\tilde{y}} - 1 \right) \right) - v(\tau + \psi(\tilde{y} - 1)) = 0$$

Let's define \tilde{y}^* as the ratio income of the *South* income of the *North*, which maximizes the growth rate in the *South*:

$$\begin{aligned}\tilde{y}^* &\stackrel{def}{\Leftrightarrow} \arg \max v \left(\tau + \psi \left(\frac{1}{\tilde{y}^*} - 1 \right) \right) \Leftrightarrow \tau + \psi \left(\frac{1}{\tilde{y}^*} - 1 \right) = a^* \Leftrightarrow \tilde{y}^* = \frac{\psi}{\psi + a^* - \tau} < 1 \\ \tau + \psi (\tilde{y}^* - 1) &= a^* - \left[1 + \frac{\psi}{\psi + (a^* - \tau)} \right] (a^* - \tau) < a^*\end{aligned}$$

where a^* is defined in equation (3.3) as the level of international aid that maximizes the growth rate of the *South*. It follows from the last equation and the assumption of $\tau < a^*$ that:

$$F(\tilde{y}^*) = v(a^*) - v(\tau + \psi(\tilde{y}^* - 1)) > v(a^*) - v(\tau) > 0 \quad (7.11)$$

It follows from (2.16) that:

$$\begin{aligned}\lim_{\tilde{y} \rightarrow 0} l_p \left(\tau + \psi \left(\frac{1}{\tilde{y}} - 1 \right) \right) &= \lim_{\frac{T}{y} \rightarrow +\infty} l_p \left(\frac{T}{y} \right) = 1 \Rightarrow \\ \lim_{\tilde{y} \rightarrow 0} v \left(\tau + \psi \left(\frac{1}{\tilde{y}} - 1 \right) \right) &= \frac{1}{\sigma} \left(\alpha A^{\frac{1}{\alpha}} \left[\frac{(1-\alpha)}{\theta B} [1-B] \right]^{\frac{1-\alpha}{\alpha}} - \delta - \rho \right) < \\ \frac{1}{\sigma} \left(\alpha A^{\frac{1}{\alpha}} \left[\frac{(1-\alpha)}{\theta B} (l_p(\tau - \psi))^{1-\beta} [1 - B (l_p(\tau - \psi))^\beta] \right]^{\frac{1-\alpha}{\alpha}} - \delta - \rho \right) &= \\ \lim_{\tilde{y} \rightarrow 0} v(\tau + \psi(\tilde{y} - 1)) &\Rightarrow \lim_{\tilde{y} \rightarrow 0} F(\tilde{y}) < 0\end{aligned} \quad (7.12)$$

where we have used the assumption that $(1-B) < (l_p(\tau - \psi))^{1-\beta} [1 - B (l_p(\tau - \psi))^\beta]$. Thus, it follows from equations (7.11), (7.12) and the continuity of $F(\cdot)$ that $\tilde{y}^{asymmetric, BGP1} \in (0, \tilde{y}^*)$ exists such that $F(\tilde{y}^{asymmetric, BGP1}) = 0$. Furthermore, $\tilde{y}^{asymmetric, BGP1} < \tilde{y}^* = \frac{\psi}{\psi + a^* - \tau} < 1$, i.e., an asymmetric BGP exists such that $\tilde{y}^{asymmetric, BGP1} < 1$.

Then, it is easy to see that:

$$\begin{aligned}F(\tilde{y}) &= v \left(\tau + \psi \left(\frac{1}{\tilde{y}} - 1 \right) \right) - v(\tau + \psi(\tilde{y} - 1)) = \\ &-v \left(\tau + \psi \left(\frac{1}{\tilde{y}} - 1 \right) \right) + v \left(\tau + \psi \left(\frac{1}{\tilde{y}} - 1 \right) \right) = -F \left(\frac{1}{\tilde{y}} \right)\end{aligned}$$

Let's define $\tilde{y}^{asymmetric, BGP2} \equiv \frac{1}{\tilde{y}^{asymmetric, BGP1}}$. It follows from the above equation that $F(\tilde{y}^{asymmetric, BGP2}) = F\left(\frac{1}{\tilde{y}^{asymmetric, BGP1}}\right) = 0$. So, $\tilde{y}^{asymmetric, BGP2}$ is another BGP. Since $\tilde{y}^{asymmetric, BGP1} < 1$ then $\tilde{y}^{asymmetric, BGP2} \equiv \frac{1}{\tilde{y}^{asymmetric, BGP1}} > 1$.

Afterwards, it follows from the definition of $F(\cdot)$ that $\tilde{y} = 1$ is a BGP, that is, the symmetric BGP, $\tilde{y}^{symmetric BGP} = 1$.

Finally, note that:

$$\tilde{y}^{asymmetric, BGP1} < 1 \Rightarrow v(\tau + \psi(\tilde{y}^{asymmetric, BGP1} - 1)) < v(\tau)$$

where we are using the assumption that $\tau < a^*$ and the fact that $v\left(\frac{\tau}{y}\right)$ is increasing when $\frac{\tau}{y} < a^*$.

7.6.2. Sufficient condition 2: $(1 - B) > (l_p(\tau - \psi))^{1-\beta} \left[1 - B(l_p(\tau - \psi))^\beta\right]$
and $\tau > a^*$

Let's define \tilde{y}^* as the ratio income of the *South* of the *North* which maximizes the growth rate in the *South*:

$$\tilde{y}^* \stackrel{def}{\Leftrightarrow} \arg \max v\left(\tau + \psi\left(\frac{1}{\tilde{y}^*} - 1\right)\right)$$

If we now define function $F(\tilde{y})$ as follows:

$$F(\tilde{y}) = v(\tau + \psi(\tilde{y} - 1)) - v\left(\tau + \psi\left(\frac{1}{\tilde{y}} - 1\right)\right) = 0$$

and we assume alternatively that $\tau > a^*$ and $(1-B) > (l_p(\tau-\psi))^{1-\beta} \left[1-B(l_p(\tau-\psi))^\beta\right]$, then it is easy to see that rest of the proof is identical to the previous case.

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