Preliminary version

On the Complementarities between Human Capital and Public Revenues: Consequences for Development*

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ABSTRACT

While there is solid empirical evidence about the significant returns to education at micro level, this relationship at macro level is more controversial, especially in developing countries. A possible explanation for this puzzle is the unproductive uses of human capital. This paper proposes a novel theory about the way in which human capital is allocated along the development process. Human capital has four possible uses: to produce private goods (private sector), to produce public goods, to collect taxes (bureaucrats) and to provide public education. At the initial stage of development, countries are characterized by low levels of human capital with a high return, which is devoted in a large portion to the private sector, due to low tax collection. The portion of human capital devoted to bureaucracy and public education grows along the transition, while the portion devoted to the private sector is declining. This may explain why the increase of human capital does not have the expected impact on production that the high private return on human capital would predict. Empirical evidence seems to support the fundamentals and implications of the model.

Keywords: Human Capital, Development, Government, Bureaucracy, Public Education

1. Introduction

One of the most intriguing puzzles in the growth and development literature is the role of the skilled labor. From a theoretical point of view there is no doubt about the positive

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impact of skilled labor in economic growth. Growth theory recognizes the contribution of skilled labor in growth process since the seminal contributions of Lucas (1988) and Romer (1990). However, the empirical macroeconomic literature is surprisingly mixed and ambiguous. While there is clear empirical evidence about the significant returns to education at micro level (see Card, 1999, for a survey), the empirical macroeconomic literature finds not only a weak relationship between skilled labor and economic performance but also shows a negative impact of skilled labor. Both, cross-sectional studies (Kyriacou, 1991, Benhabib and Spiegel, 1994, Nonneman and Vanhoudt, 1996, Pritchett, 2001) as more recent panel data studies (Kumar, 2006, Bond, Hoeffler and Temple, 2001, Caselli, Esquivel and Lefort, 1996; Islam, 1995) report insignificant or negative impact of skilled labor on economic growth, constituting a puzzle which has attracted the interest of many researchers in Growth and Economic Development.

This empirical puzzle is revealed even more interesting when it is considered jointly the observation that the return on education is not the same among countries. Psacharopoulos (1994), Psacharopoulos and Patrinos (2004) and Strauss and Duncan (1995) evidence that the return to education in developing countries is extremely high and it declines with per capita income, what would imply developing countries growing at faster speed than developed countries. Nevertheless, it is also well documented (see Easterly 1994, 2001), that such a high return to education do not always lead to a successful growth process. This suggests that the relationship between skilled labor and growth performance is even weaker in developing countries.

This paper offers a new explanation to understand why there exists a weak relationship between skilled labor and economic growth in the first stages of development. The aim of this paper is to build a theory which explains how human capital is allocated in the economy during the development process. This framework would allow us to account for the main features related to the evolution of the human capital when economies are growing.

The theory that this paper proposes is based on three other stylized facts. First, governments in developing countries have much difficulty to raise public revenues. Second, an important portion of workers with skilled labor engage the public sector in developing countries. Third, in spite of there exists the concern about some enrollment of educated bureaucrats in rent-seeking activities, there is also evidence of generating positive effects. More precisely, we build a model where public revenues determine the amount of public expenditure on education, which it turn, it is an important factor to produce skilled labor. In the model the skilled labor has four uses: to produce private and public goods, to collect taxes (bureaucracy) and to produce skilled labor throughout the public education system (teachers). The level of tax collection depends positively on the size of the bureaucracy. A portion of public revenues is used to produce the public good and the remaining portion is used to pay bureaucrats and teachers. Thus, a feedback process arises: a higher level of skilled labor implies more efficient bureaucrats, who collect more taxes that are used to finance a public expenditure on education which in turn promotes skilled labor. Insofar skilled labor is growing throughout the transition to the steady state, the amount of collected taxes, the public expenditure on education and the size of bureaucracy also increase. There is a certain level of skilled labor in which the bureaucracy reachs the point in which the tax collection is maximized, and the effective tax rate coincides with the statutory tax rate. After this level, the bureaucracy sector remains stationary and any increment in the skilled labor is devoted only to the provision of education and to produce goods.

Notice that under certain conditions, the feed-back process described above may generate multiple steady state. When the per capita skilled labor is low, the scarcity of the skilled labor implies high private returns on education, encouraging the skilled labor accumulation. Nevertheless, on the other hand, there is also a low level of tax collection which implies a low expenditure in public education the economy. Given the fact that public education is a significant input in the production function of skilled labor, the low provision of education implies low return of skilled labor. This second mechanism on the return of skilled labor reduces partially the high private incentive to accumulate skilled labor, decelerating the skilled labor accumulation. The net effect on the accumulation of skilled labor will depend on the relative size of these two offsetting mechanisms. Thus, a poverty trap characterized by a "vicious circle" may exists: public revenues are low because skilled labor and income are low due to the scarcity of public expenditure on education, which can not be raised due to the low public revenues. Nevertheless we disregard this cases and concentrate in the transition to an unique steady state.

The first contribution of the paper is providing an explanation of the evolution of the human capital along the development process which is consistent with empirical facts. The model predicts a structural change throughout the transition which involves a reallocation of skilled labor to different uses: production of private and public goods, bureaucracy to collect taxes and the public education. In the first stages of development, when the level of per capita skilled labor is low, the amount of skilled labor devoted to bureaucracy and public education increases throughout the transition, estimulating the accumulation of skilled labor and so generating again increases in the amount of collected taxes, the public expenditure on education and the size of bureaucracy. On the contrary, the amount of skilled labor allocated in the production of goods shows an unclear pattern. Notice that if the government's requierement of human capital to provide the economy of bureaucrats and teachers is high enough, the amount of human capital devoted to production of final goods even might decrease. Afterwards, once the economy reaches the level of skilled labor in which the gap between the effective and the statutory tax rates is closed, the bureaucracy sector remains stationary and the skilled labor is devoted enterely to public education and to produce goods. This stage of development in which the economy converges to the steady state is characterized by an increase of the share of skilled labor devoted to the production of goods.

The second contribution of the paper is offering an explanation to understand the cause of the weak relationship between skilled labor and economic performance documented by the empirical research. This paper suggest that a possible reason may be the reallocation of skilled labor toward bureaucracy and the public education system during the first stages of development. This reallocation is important and necessary, but detract resources from producing good. The fact that not all the skilled labor is devoted to public education or to produce goods may explains why, at least temporally, the skilled labor shows a low impact in the economy. In this framework, social returns of skilled labor would be obtained with a delay over time.

Third, the paper highlights the importance of the allocation of tax revenues in its different uses, like expenditure in public good or expenditure in the public educational system, and how this allocation do not only have temporally effects throughout the transition, but also in the long run. We have proved that if the share of public revenues which is devoted to provide the public good decreases and consequently, it increases the amount of public resources devoted to finance the public education, then the levels of consumption and skilled labor increase in the steady state. Consequently, countries with the same initial skilled labor level and identical characteristics could have different structural process and economic results depending on how much they decide invest in skilled labor. This observation is in the line of the argues of Temple (1999), which finds that the impact of skilled labor on the economy is different among countries.

Finally, the model is based on the well documented fact that the higher is the education of the bureaucracy, the better is the economic performance of the economy. However, it is worth to notice that the mechanism proposed in the paper does not excludes the possibility that some part of the bureaucrats may not be involved in unproductive or rent-seeking activities (see Blackburn, Bose, and Haque 2006, Mauro, 2004). In the context of the paper, this kind of corruption would imply less effective tax collection. We also have proved that an improvement in the technology of the bureaucracy, this is, a gain in the efficiency which implies that the level of tax collection increases with the same number of bureaucrats, produces a reallocation of skilled labor form bureaucracy to the private sector. Interestingly, while the total amount of skilled labor decreases in the new steady state, consumption and per capita income reach higher levels. Therefore, according to the prediction of the model, the impact of skilled labor in the production may be delayed by a low level of the bureaucracy efficiency.

The rest of this paper is organized as follows. Section 2 discusses the related literatures. Section 3 describes main empirical facts on which the model is based on. Section 4 presents the basic elements of a model where the skilled labor has three different uses: to produce goods, to produce education (teachers) and to produce bureaucrats. Section 5 analyses the behavior and decisions of agents in the economy. Section 6 characterizes the steady state of the economy. Section 7 describes the dynamics of the economy. Section 8 shows the results of several experiments and finally, section 9 summarizes. All the proofs are included in the Appendix.

2. Related literature

There have been many attempts in the literature to understand the empirical evidence which documents the coexistence of high returns of education at micro level with a weak relationship between skilled labor and economic growth.

One strand of the literature has focused on the possible existence of technical and empirical problems in measuring the effects of the skilled labor. Cohen and Soto (2007) claim that part of the reason behind the evidenced reduced role of human capital on economic growth is due to the measurement of human capital, both conceptually and empirically. In this regard, De la Fuente and Domenech (2002, 2006), for a sample of 21 OECD countries, show that the existing human capital data is fairly unreliable. Also Krueger and Lindahl (2001) show that variable years of schooling contains little information in many cases. More recently Soto (2009) states that another problem IS obtaining reliable estimates of the social return on schooling given the estimation problems found in the literature.

However, another set of researchers have attempted to provide explanations for the insignificant or negative impact of skilled labor on economic growth. They share the vision of Temple (1999), which argues that for solving this puzzle it is needed to accept that the impact of skilled labor on growth has not been the same among countries. One strand of the literature highlights that conventional factors of production such as physical capital, skilled labor and technology are not the only driving mechanisms behind the growth performance (Easterly and Levine, 2002, Acemoglu, 2009). From this strand, the importance of institutions and corruption are suggested as ones of the main causes of the economic growth (see Hall and Jones, 1999; Acemoglu et al., 2001 for the institutional approach and Mauro, 2004, Blackburn, Bose, and Haque 2006 for the corruption one). Other strand of the literature focuses on characteristics of the skilled labor sector. North (1990) points out the possibility of an allocation problem in skilled labor: if the demand for skilled labor comes in some extent from individually remunerative yet socially wasteful activities, in this case, the education could rise the wage of each individual (producing the micro evidence), even while increase in average education would cause aggregate output to stagnate or fall (producing the macro evidence). Princhett (2001) also argues that if schooling quality may be so low that it does not raise cognitive skills or productivity (producing the macro evidence), this result could even be consistent with higher private wages in the case of education serves as a signal to employers of some innate ability. In summary, while all of these models offer explanations based on the existence of improductive services of the human capital, our model proposes a new explanation which suggests that all services are productive but there exists a delay in order to see their effects in the output of the economy.

3. The basis of the model: empirical facts

This paper investigates on the relationship between the human capital and the economic growth along the development process. Empirical literature on development has identified many facts which characterize the behaviour of the economies during development process. In this section we present the most relevant stylized facts regarding the main features which are the basis of our theory:

(i) *The level of tax collection*: One of the main differences between developed and developing countries can be found in the level of tax collection, with developing countries showing reduced levels. Easterly and Rebelo (1993.a, 1993.b) and Gordon and Li (2009) find that the important differences in per capita GDP tax revenues between developed and developing countries cannot be explained by differences in statutory tax rates. While the maximum statutory personal income tax rate on average in developed countries is 1.23 times than in developing countries, the ratio between personal income tax revenue and GDP is 5.3 larger. According to their findings, income tax evasion is an important phenomenon, in particular for developing countries. These number reveal the difficulty of governments to raise taxes in developing countries.

(ii) *The number of skilled workers in the public sector*: There exists a generalized perception of excessive public employment in developing countries. Rama (1999) notices that financial aid by the IMF or World Bank to these countries is frequently conditioned to public sector downsizing clauses. More importantly, many anecdotical evidence support the fact that a large fraction of workers hired by the goverment are educated (see Schmitt, 2010). For example, Murphy et al. (1991) note that in many African countries educated individuals apply for jobs in the public sector attracted by rents derived from bribery. Gelb et al. (1991) for a sample of developing countries find evidence that the public employment has grown faster than employment in any other sector. Pritchett (2001) cites the case of Egypt where the government sector employed seventy percent of university graduates at the end of the nineties. More recently Banerjee (2006) and Schündeln and Playforth (2014) find strong evidence that supports the observation that the public sector in India has attracted and captured a many well-educated individuals.

(iii) *The efficiency of bureucrats:* In spite of there exists the concern about some enrollment of educated bureaucrats in rent-seeking activities, there is also strong evidence of generating positive effects. As Pritchett (2001) argues "the question is not whether the educated labor flows into the government so much as what the educated labor does once it is in the government". A report by the World Bank (1993) attributed the economic success of the East Asian 'Tigers' to their high quality of bureaucracy. A year later, the World Bank (1994) finds that the most successful of developing countries had highly educated bureaucrats. The level of the skilled workers in the public sector has been identified as a necessary condition for an economy to be successfully achieve economic growth (Evans, 1995 and Rauch and Evans, 2000).

4. Model

4.1. Technology in the production sector

Time is continuous and endless, and it is index by $t \in \Re_+$. There is two goods in the economy: the private consumption good, denoted in per capita terms as c, and the public provided good, denoted in per capita terms as g. There are two production factors: skilled labor, denoted in per capita terms as h, and raw (unskilled) labor, denoted in per capita terms as l. There are a continuum of workers which are either skilled or unskilled, having each of type of worker one unit of his type of labor. Production of goods can be devoted to private and public consumption and to investment in skilled labor. The production technology of this good is given by the Cobb-Douglas production function:

$$y(t) = Al(t)^{1-\alpha} h_y(t)^{\alpha} = c(t) + g(t)$$

where $y(t)$ denotes per capita production of goods, $h_y(t)$ the per capita skilled labor devoted to the production of goods and $l(t)$ the unskilled labor at t; $A \in R_{++}$ is the total factor productivity and $\alpha \in (0,1)$ is the share of the skilled labor, $c(t)$ denotes per capita consumption and $g(t)$ denotes per capital public good provided by the government and consumed by households.

4.2. Demography and education technology

Fertility rate is constant and denoted by $b > 0$. Agents survive next period with probability $1 - m$, being $m \in (0, 1)$ the mortality rate. This implies that population grows at a constant rate $n \equiv b - m \geq 0$. Agents born unskilled, if they want to become skilled workers they have to be involved in an education process which is costly. Individuals have to devote their whole time to education during one period. We will call the agent being educated student, we will denote the per capita amount of students as $s(t)$. Furthermore, a student reachs human capital and becomes skilled worker with probability $\mu(z)$, where $z \equiv h_h/s$ denotes the the ratio teachers-student and h_h is the amount of per capita teachers. Teachers are provided by the government. Thus, the amount of skilled workers behaves according with the following law of motion:

$$\dot{H}(t) = \mu(z(t)) S(t) - mH(t)$$

The above equation means the total amount of skilled workers, $H(t)$, increases with the amount of unskilled workers that reach human capital through the education process, $\mu(z(t)) S(t)$, where $S(t)$ is the total amount of students, and decrease with the number of skilled workers that die, $mH(t)$. If we write the above equation in per capita terms we get:

$$\dot{h}(t) = \mu(z(t)) s(t) - bh(t) \tag{1}$$

We also assume that the probability that a student become a skilled worker is as follows:

$$\mu(z) = \begin{cases} z^{\xi} & \text{if } z \leq 1 \\ 1 & \text{if } z > 1 \end{cases}$$

Note that $\mu(z)$ is decreasing in the parameter ξ and that when $\xi = 0$ then $\mu(z) = 1$. Thus, we will consider that ξ is an inverse index of the quality of the educational system, the lower ξ the better the performance of the educational system.

4.3. Household

There are many identical households, each of them with a continuum of agent of measure $N(t)$, which evolves according with the birth and the mortality rate:

$$\dot{N}(t) = bN(t) - mN(t) = (b - m)N(t) = nN(t)$$

Households are composed by skilled workers, unskilled workers and students, i.e.,

$$N(t) = H(t) + L(t) + S(t)$$

In per capita terms:

$$h(t) + l(t) + s(t) = 1$$

The preferences of a household are given by a time separable utility function:

$$\int_0^\infty N(t)\left[u\left(c(t)\right) + \phi u\left(g(t)\right)\right] e^{-\rho t} = N_0 \int_0^\infty \left[u\left(c(t)\right) + \phi u\left(g(t)\right)\right] e^{-(\rho-n)t}$$

where $c(t)$ denotes the household's per capita consumption at period t, $g(t)$ denotes the household's per capita public good at period t, $\rho > n$ denotes the utility discount rate and, function $u(.)$ is the CES utility function:

$$u(x) = \begin{cases} \frac{x^{1-\sigma}}{1-\sigma} & \text{if } \sigma \in (0,1) \cup (1,+\infty) \\ \ln x & \text{if } \sigma = 1 \end{cases}$$

where $x = \{c, g\}$.

4.4. Government

The government provides a public good to the households and hires a certain amount of human capital as teachers to produce more human capital. In order to finance these expenditures, the government fix a "statutory" tax rate, $\overline{\tau}$, on the average earnings running from the human capital activities. However, in this model the government needs to hire bureaucrats to collect taxes. If there is no any bureaucracy to manage and control the tax collection, individuals would not pay any tax. Thus, the effective tax rate that individuals are paying depends positively on the bureaucrats that the government hires. There is a technology which translates the bureaucracy efforts in effective public revenues. In particular, the effective tax rate which is paid and so it produces public revenues in period t is as follows:

$$\tau\left(h_b(t)\right) = \begin{cases} \Gamma\left(h_b(t)\right)^\gamma & \text{if } h_b(t) < \overline{h}_b \\ \overline{\tau} & \text{if } h_b(t) \geq \overline{h}_b \end{cases} \qquad \overline{h}_b \equiv \left(\frac{\overline{\tau}}{\Gamma}\right)^{\frac{1}{\gamma}} \tag{2}$$

$$\Leftrightarrow \tau\left(h_b(t)\right) = \min\left\{\Gamma\left(h_b(t)\right)^\gamma, \overline{\tau}\right\} \tag{3}$$

where $\tau\left(h_b(t)\right)$ denotes the effective tax rate which is paid by individuals in period t, and $h_b(t)$ is the amount of per capita skilled labor devoted to the bureaucrat sector (number of bureaucrats), $\Gamma > 0$ and $\gamma \in (0,1)$. It is assumed that the higher is the number of the bureaucrats assigned to manage the tax collection, the higher is the effective tax rate and so the higher is the amount of public revenues. There is a maximum number of bureaucrats, $\overline{h}_b$, that makes the effective tax rate, $\tau\left(h_b(t)\right)$, equal to the statutory tax rate, $\overline{\tau}$.

Wages that government pays to bureaucrats and teachers are not controlled by it. Because of the human capital is assumed to be perfectly substitutable among sectors and there is perfect competition, then, wages equates among sectors and they are the same that the wages paid in the production sector. We denote by w_h the wage of the skilled labor.

Therefore, the government budget constraint is as follows:

$$\tau\left(h_b\right) w_h \left(h_y + h_h + h_b\right) = g + w_h \left(h_h + h_b\right) + tr \tag{4}$$

The left side of the equation is the total public revenues of the government in per capita terms that come from the taxation over the human capital income. Per capita public revenues are defined by the effective tax rate multiplied by the per capita skilled labor income. The right side of the equation are the four expenditures of the government: i) the per capita public good provided to households, g; ii) the per capita expenditure in the public education system, $w_h h_h$, that is, the wages paid to teachers and; iii) the per capita wages paid to bureaucrats, $w_b h_b$ and; iv) the per capita amount of transfers devoted to households, tr. For simplicity, we assume that the government devote a fraction, $\lambda \in (0,1)$, of the public revenues for hiring teachers, and a fraction $\varphi \in (0,1)$ to provide the public good. The remaining tax revenues are devoted to pay bureaucrats and to transfer payments to households, this is,

$$\lambda\tau\left(h_b\right) w_h h = w_h h_h \tag{5}$$

$$\varphi\tau\left(h_b\right) w_h h = g \tag{6}$$

$$\left(1-\lambda-\varphi\right)\tau\left(h_b\right) w_h h = w_h h_b + tr \tag{7}$$

We want to focus our attention on the dynamics of the skilled labor throughout the transition to the steady state equilibrium of the economy and, specially, on the skilled labor reallocation among the different sectors of the economy. For this reason, we simplify the model adoptting the assumption that the unique reproducible factor is the skilled labor. This simplification assumption is perfectly justified, since the introduction of another reproducible factor would not alter at all the reallocation mechanisms of the skilled labor along the transition.[1].

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[1]To this respect, Bucci and Segre (2011) consider the same assumption to study the effects of the interaction between cultural and human capital.

5. Agents' decisions

5.1. Firms

Firms behave competitively and hire the amount of workers and skilled labor that maximize their profits:

$$\max_{L(t),H_y(t)} AL(t)^{1-\alpha}H_y(t)^{\alpha} - w_h(t)H_y(t) - w(t)L(t) \tag{8}$$

where $L(t)$ and $H_y(t)$ denotes the amount of unskilled labor and skilled labor hired by the firm at period t and $w(t)$ denotes the wage of the unskilled labor at period t. The first order conditions of the above problem are:

$$\begin{aligned} \alpha A\left(\frac{L(t)}{H_y(t)}\right)^{1-\alpha} &= w_h(t) \\ (1-\alpha)\,A\left(\frac{H_y(t)}{L(t)}\right)^{\alpha} &= w(t) \end{aligned}$$

That is, firms hire a factor until the point in which the marginal productivity of such factor is equal to its price. These first order conditions may be rewritten in per capita terms:

$$\alpha A\left(\frac{l(t)}{h_y(t)}\right)^{1-\alpha} = w_h(t) \tag{9}$$

$$(1-\alpha)\,A\left(\frac{h_y(t)}{l(t)}\right)^{\alpha} = w(t) \tag{10}$$

5.2. Households

The households' optimization problem is as follows:

$$\max_{\{c(t)\}_{t=0}^{\infty}} \int_0^{\infty} \left[u\left(c(t)\right) + v\left(g(t)\right)\right] \left(\frac{1+n}{1+\rho}\right)^t dt \tag{11}$$

$$c(t) = w_h(t)\left(1-\tau(t)\right)h(t) + w(t)\left(1-h(t)-s(t)\right) + tr(t) \tag{12}$$

$$\dot{h}\left(t\right) = \mu\left(z(t)\right)s\left(t\right) - bh\left(t\right) \tag{13}$$

$$h\left(0\right) > 0$$

Households maximize their utility subject to: (i) their budget constraint: the expenditure in consumption $c(t)$ should be equal to their disposable income that come from skilled labor

$w_h(t)h(t)$ and unskilled labor $w(t)\,(1-h(t)-s(t))$, minus taxes $\tau(t)w_h(t)h\,(t)$ plus transfers $tr(t)$; (ii) the accumulation equation of skilled labor. The Euler Equation and the transversality condition associated to the households' optimization problem are:

$$\frac{\dot{c}\,(t)}{c\,(t)} = \frac{1}{\sigma(c)}\left[\frac{w_h\,(t)\,(1-\tau(t))-w\,(t)}{p_h(t)}+\frac{\dot{p}_h(t)}{p_h(t)}-m-\rho\right] \tag{14}$$

$$\lim_{t\to+\infty} u'(c(t))e^{(\rho-n)t}p_h(t)h(t) = 0 \tag{15}$$

where $p_h(t) = \frac{w(t)}{\mu(z(t))}$ is the marginal cost of producing one unit of skilled labor. Such cost is equal to the amount of unskilled labor required to produce one unit of skilled labor, $1/\mu\,(z(t))$, multiplied by the prices of use (the opportunity cost) of the unskilled labor $w(t)$. The first of the above conditions is the Euler equation. The speed at which consumption grows it displays the standard features: depends positively on the return of investment in skilled labor, $\frac{w_h(t)(1-\tau(t))-w(t)}{p_h(t)}+\frac{\dot{p}_h(t)}{p_h(t)}$, and negatively on the patient rate of the household, ρ, and the depreciation of the human capital measured by the mortality rate, m. Notice that the return of the human capital takes the form of the return of an asset: the first part, $\frac{w_h(t)(1-\tau(t))-w(t)}{p_h(t)}$, captures the direct return of investment in human capital and the second part, $\frac{\dot{p}_h(t)}{p_h(t)}$, measures the possible gains of capitalization derived from changes in the price of human capital. The difference with the standard case is that here individuals care about the ex ante return to human capital, $\mu(z(t))\frac{w_h(t)(1-\tau(t))-w(t)}{w(t)}$, instead the ex post return, $\frac{w_h(t)(1-\tau(t))-w(t)}{w(t)}$. In other words, individuals are aware that there exist a certain probability of not acquiring the human capital, $\mu(z(t))$, when they are inverting in it. Finally, the more concave the utility function (the higher $\sigma(c)$), the smoother the consumption path. The second equation is the standard transversality condition.

5.3. Government fiscal policy

In this economy, the government wishes collect taxes to produce a public good, to finance transfers to households and to provide teachers. However, to do that the government needs to hire bureaucrats in order to manage the tax collection. Therefore, the government maximizes the amount of public revenues minus the cost in bureaucrats needed to produce it. Once bureaucrats are hired and public resources are generated, the government can provide the public good, pay transfers and hire teachers to increase the private production of human capital. Thus, given the tax rate, $\overline{\tau}$, over the earnings from skilled labor, and the total amount of skilled labor, the government has to decide each period how many bureaucrats, $h_b(t)$, hires at current wages in order to obtain the maximum amount of public revenues. In other words, the government chooses the number of bureaucrats to hire, in order to maximize the total amount of per capita taxes paid by individuals, $T(t)$.

Therefore, the problem of the government in period t is formalized as follows:

$$\max_{h_b} T(t) - w_h(t)h_b(t)$$

where

$$T(t) = \tau\left(h_b(t)\right) w_h(t)h(t) = \min\left\{\Gamma\left(h_b(t)\right)^{\gamma}, \overline{\tau}\right\} w_h(t)h(t) \tag{16}$$

Then, the problem of the government results as:

$$\max_{h_b} \tau\left(h_b(t)\right) h(t) - h_b(t) \tag{17}$$

The solution of the problem is the optimal amount of bureaucrats:

$$h_b(h(t)) = \begin{cases} (\gamma\Gamma)^{\frac{1}{1-\gamma}} h(t)^{\frac{1}{1-\gamma}} & \text{if } h(t) < \overline{h} \\ \overline{h}_b & \text{if } h(t) \geq \overline{h} \end{cases}; \quad \overline{h} = \frac{\overline{\tau}^{\frac{1-\gamma}{\gamma}}}{\gamma\Gamma^{\frac{1}{\gamma}}}; \quad \overline{h}_b = \frac{\overline{\tau}^{\frac{1}{\gamma}}}{\Gamma^{\frac{1}{\gamma}}} \tag{18}$$

$$T(h(t)) = \begin{cases} w_h(t)\left(\gamma^{\gamma}\Gamma\right)^{\frac{1}{1-\gamma}} h(t)^{\frac{1}{1-\gamma}} & \text{if } h(t) < \overline{h} \\ \overline{\tau} w_h(t)h(t) & \text{if } h(t) \geq \overline{h} \end{cases}; \tag{19}$$

Notice that the share of the tax collection devoted to bureaucrats is equal to γ. Thus, in order to guarantee the existence of non negative transfer payments we assume that the fraction of taxes devoted to bureaucrats, γ, plus the fraction devoted to teachers, λ, and the fraction devoted to government expenditure, φ, are together equal or smaller than one: $\gamma + \lambda + \varphi \leq 1$.

Finally, the transfer payments and the government expenditures would be as follows:

$$g(h(t), w_h(t)) = \begin{cases} \varphi w_h(t)\left(\gamma^{\gamma}\Gamma\right)^{\frac{1}{1-\gamma}} h(t)^{\frac{1}{1-\gamma}} & \text{if } h(t) < \overline{h} \\ \varphi\overline{\tau} w_h(t)h(t) & \text{if } h(t) \geq \overline{h} \end{cases}; \tag{20}$$

$$tr(h(t), w_h(t)) = \begin{cases} (1-\gamma-\lambda-\varphi)\, w_h(t)\left(\gamma^{\gamma}\Gamma\right)^{\frac{1}{1-\gamma}} h(t)^{\frac{1}{1-\gamma}} & \text{if } h(t) < \overline{h} \\ w_h(t)\left[(1-\lambda-\varphi)\,\overline{\tau} h(t) - \overline{h}_b\right] & \text{if } h(t) \geq \overline{h} \end{cases}; \tag{21}$$

6. The dynamics of the allocation of human capital among sectors

Once we determine the optimal amount of bureaucrats (eq. (18)), h_b, we can obtain the amount of human capital which is allocated in the remaining uses. We first obtain the amount of human capital which is devoted to the public education program. To do that, we substitute the amount of bureaucrats, eq. (18) in eq. (2) and we obtain the effective tax rate in the economy. Then, with the tax rate, we find the amount of teachers, h_h, using equation eq. (5). Finally, we obtain the amount of human capital which is devoted to the production of goods, h_y, as the remainig amout of human capital after the two previous uses. The dynamics of the allocation of human capital in its three possible uses, bureaucracy, teaching or production, is described by (see appendix):

$$h_b(h(t)) = \begin{cases} (\gamma\Gamma)^{\frac{1}{1-\gamma}} h(t)^{\frac{1}{1-\gamma}} & \text{if } h(t) < \overline{h} \\ \overline{h}_b & \text{if } h(t) \geq \overline{h} \end{cases} \tag{22}$$

$$h_h(h(t)) = \begin{cases} \lambda\,(\gamma^{\gamma}\Gamma)^{\frac{1}{1-\gamma}} h(t)^{\frac{1}{1-\gamma}} & \text{if } h(t) < \overline{h} \\ \overline{\tau}\lambda h(t) & \text{if } h(t) \geq \overline{h} \end{cases} \tag{23}$$

$$h_y(h(t)) = \begin{cases} h(t) - (\gamma + \lambda)\,(\gamma^{\gamma}\Gamma)^{\frac{1}{1-\gamma}} h(t)^{\frac{1}{1-\gamma}} & \text{if } h(t) < \overline{h} \\ (1 - \overline{\tau}\lambda)\,h(t) - \overline{h}_b & \text{if } h(t) \geq \overline{h} \end{cases} \tag{24}$$

The above equations are displayed in figure 1. The evolution of the of the three different uses of human capital depends on the evolution of the effective tax rate. The effective tax rate is an increasing function of the per capita human capital until reaching the certain level, $\overline{h}$, in which the effective tax rate coincides with the statutory tax rate $\overline{\tau}$. Beyond this threshold, the effective tax rate is constant and equal to the statutory tax rate. When the per capita human capital remains below the threshold level $\overline{h}$, the evolution of the effective tax rate implies that all uses of human capital in the public sector (bureaucrats, teachers and human capital used in the production of the public good) rise with per capita human capital, while the share of human capital used to produce private goods might show a hump-shaped pattern. Since the amount of human capital devoted to bureaucrats and teachers increases faster than the total amount of human capital in the economy, the remaining amount of human capital which is devoted to the production of private goods increases but a decreasing rate. However, if the requirement of the government for human capital is growing highly enough, it might ocurr that there will not chance to continuing allocating human capital to the private sector, declining the amount of human capital in the production of goods. In this case, there exists a drain of human capital in the economy from the private sector to the public sector. Figure 1 illustrates this case.

7. The definition of equilibrium

The equilibrium definition is standard: equilibrium occurs when agents maximize their objective functions and markets clear. Since all households and firms are alike, we may define equilibrium in per capita terms.

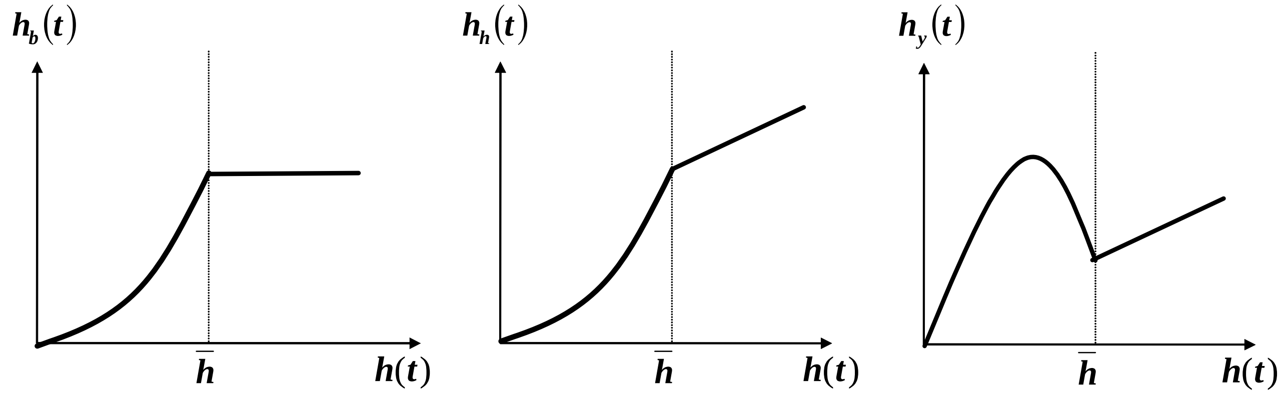


Fig. 1.— Evolution of different uses of human capital

Definition 1 *Given the initial condition* h_0*, a competitive equilibrium is an allocation* $\{c(t), s(t), h(t), h_y(t), l(t), h_b(t), h_h(t), g(t), tr(t)\}_{t=0}^{\infty}$ *and a vector of prices* $\{w(t), w_h(t)\}_{t=0}^{\infty}$ *such that* $\forall t$:

- Households maximize their utility, that is, $\{c(t), s(t), h(t)\}_{t=0}^{\infty}$ is the solution of the household's maximization problem (11).
- Firms maximize profits, that is, $h_y(t), l(t)$ is the solution of the optimization problem of firms (8).
- The government chooses the amount of skilled labor devoted to bureaucracy, $h_b(t)$, which maximizes the public revenues net of bureaucratic expenditure (17) and chooses the amount of skilled labor devoted to the public educational system (teachers) $h_h(t)$, the expenditure in the public good $g(t)$ and the transfer payments according with the fiscal policies rules (5), (6) and (7).
- Skilled labor market clears: $h(t) = h_h(t) + h_b(t) + h_y(t)$.
- Unskilled labor market clears: $l(t) = 1 - h(t) - s(t)$.
- Goods market clears: $Ah_y(t)^{\alpha} l(t)^{1-\alpha} = c(t) + g(t)$

Definition 2 *Steady state equilibrium is an equilibrium in which both the allocation and the vector of prices always remain constant over time.*

8. Dynamic behavior

The dynamic behavior of this economy could be characterized by the dynamics of the consumption and the skilled labor variables. We now proceed to define the dynamic system of the economy.

8.1. Dynamic system

The dynamic system of this economy consists of the skilled labor accumulation equation (13), the Euler equation (14) and the transversality condition equation (15), this is,

$$\begin{aligned}
\dot{h}(t) &= \mu\left(\frac{h_h(h(t))}{s(t)}\right)s(t) - bh(t)\\
\frac{\dot{c}(h(t),s(t))}{c(h(t),s(t))} &= \frac{1}{\sigma}\left[\frac{w_h(h(t),s(t))\,(1\text{-}\tau(h_b(h(t))))\text{-}w(h(t),s(t))}{p_h(h(t),s(t))} + \frac{\dot{p}_h(h(t),s(t))}{p_h(h(t),s(t))} - m - \rho\right]\\
&\lim_{t\to+\infty} u'(c(h(t),s(t)))\,e^{-\rho t}p_h(h(t),s(t))\,h(t) = 0
\end{aligned}$$

where $w_h(h,s)$ is the marginal product of skilled labor in the production sector, which coincides at equilibrium with its wage; $w(h,s)$, is the marginal product of unskilled labor in the production sector, which coincides at equilibrium with its wage; $p_h(h,s)$ is the marginal cost of the production of one unit of skilled labor; and $c(h,s)$ is the per capita household's consumption after tax/transfers income, this is,

$$\begin{aligned}
w(h,s) &= (1-\alpha)A\left(\frac{h_y(h)}{1-h-s}\right)^{\alpha}; \quad w_h(h,s) = \alpha A\left(\frac{1-h-s}{h_y(h)}\right)^{1-\alpha}; \quad p_h(h,s) = \frac{w(h,s)}{\mu\left(\frac{h_h(h)}{s}\right)}\\
c(h,s) &= w_h(h,s)\left[1-\tau(h_b(h))\right]h + w(h,s)(1-h-s) + tr(w_h(h,s),h)
\end{aligned}$$

The above dynamic system may be rewritten as follows:

$$\dot{h}(t) = F_h(h(t),s(t)) \tag{25}$$

$$\dot{s}(t) = F_s(h(t),s(t)) \tag{26}$$

where:

$$F_h(h,s) = \mu\left(\frac{h_h(h)}{s}\right)s - bh$$

$$F_s(h,s) = \frac{\left[\frac{\frac{\partial c(h,s)}{\partial h}}{c(h,s)} - \frac{1}{\sigma}\frac{\frac{\partial p_h(h,s)}{\partial h}}{p_h(h,s)}\right]F_h(h,s) - \frac{1}{\sigma}\left[\frac{w_h(h,s)(1-\tau(h_b(h)))-w(h,s)}{p_h(h,s)} - m - \rho\right]}{\frac{1}{\sigma}\frac{\frac{\partial p_h(h,s)}{\partial s}}{p_h(h,s)} - \frac{\frac{\partial c(h,s)}{\partial s}}{c(h,s)}}$$

Proposition 3 *If $\xi < 1-\gamma$, $\lambda\overline{\tau} > b$ and $\Gamma > \overline{\Gamma}$, where $\overline{\Gamma}$ is a constant defined in the appendix, then there is a unique steady state equilibrium, h^{ss}. Furthermore, $h^{ss} > \overline{h}$.*

The above proposition says that if the productivity of the education system is high enough (ξ is small enough), the productivity of the tax collection technology is high enough ($\Gamma > \overline{\Gamma}$) and the portion of human capital devoted to education is large enough ($\lambda\overline{\tau}$ is large enough), then there are not poverty traps. Instead, there exists a unique steady state in which the effective tax rate coincides with the statutory one. This proposition finds that one key factor in order to avoid the existence of the poverty traps is the quality of the eduction system. If the quality of the education system is too low, there is not incentives to invest in education when the starting level of human capital is low. In this case, poverty traps appear. As a matter of fact, if $\xi > 1-\gamma$, the economy may converge to the trivial steady state, in which the amount of human capital is zero. In this case, the low quality of the public system and so, the reduced probability of a student become a skilled worker, imply that the economy is not able even to replace the amount of skilled worked which depreciates each period. Thus, the per capita amount of human capital declines each period converging to zero.

We will concentrate in the case in which there is a unique steady state, and the effective tax rate coincides with the statutory one at the steady state. Thus we assume from now on that $\xi < 1-\gamma$, $\lambda\overline{\tau} > b$ and $\Gamma > \overline{\Gamma}$.

Proposition 4 *If $\xi < \alpha$ then there exist $\overline{\sigma} > 0$ such that if $\sigma > \overline{\sigma}$ then the steady state equilibrium is a saddle point and the number of students increase along the converging path to the steady state when $h(t) < h^{ss}$.*

The saddle point dynamic implies there is a unique path which converges to the steady state. This means, that given the initial level of per capita skilled labor, there is only one equilibrium trajectory which ends at the steady state. Notice that in this model while the amount of skilled labor evolves along time also, it also does the amount of students and the amount of non-skilled labor. However, the dynamics of the students and non-skilled labor are no trivial. Imagine that the initial stock of skilled labor, $h(0)$, is below than the the steady state level, $h(0) < h^{ss}$, and so, due to the relative scarcity of the human capital, the return of education is high. To determine the relationship between human capital and the amount of students we have to consider two effects: a substitution effect and a wealth effect. Insofar, countries accumulate human capital, the return of human capital decreases, reducing in turn, the incentive to having more students in the economy. Thus, a substitution effect would imply a decrease in the number of students. Simultaneously, when countries own more human capital and can afford for higher levels of consumption, they would tend to have more students since they would like to enjoy for a higher future levels of consumption. Thus, a wealth effect would imply an increase in the number of students. The resulting net effect would depend on the relative size of those two effects. However, the relevant case from the empirical point of view is the one in which the number of students increases during the development process. So, if we want the model has the property of that the number of students increases during the transition while countries accumulate human capital, the substitution effect should not be too large. This is the reason why we will concentrate in the case in which the elasticity of substitution is small enough, $\frac{1}{\sigma} < \frac{1}{\overline{\sigma}}$, this is, when the parameter sigma is large enough, $\sigma > \overline{\sigma}$. Finally, if the amount of skilled labor and the number of students increase throughout the transtition then, as a residual, the amount of raw labor will decrease.

Phase diagram in figure 2 shows that the dynamic behavior of the economy is characterized by the typical saddle point dynamic. In the case considered, $\sigma > \overline{\sigma}$, when the initial amount of per capita skilled labor is lower than the steady state level, the number of students grows throughout the equilibrium path, converging to its steady state level. When the amount of per capita skilled labor is larger than the steady state level the opposite happens.

8.2. The dynamics of the allocation of human capital among sectors

We now analyze the allocation of skilled labor among different sectors throughout the transition to the steady state. When the starting amount of per capita skilled labor is below the steady state level and the skilled labor threshold, $\overline{h}$, a "structural change" arises: insofar countries are accumulating skilled labor, the size of the bureaucracy increases, and consequently the effective tax rate rises as well (see eq. (18) and eq. (2) respectively). The increase in the effective tax rate rises tax revenues and this allows to the government to hire more skilled labor that will be devoted to the public education system (teachers) and

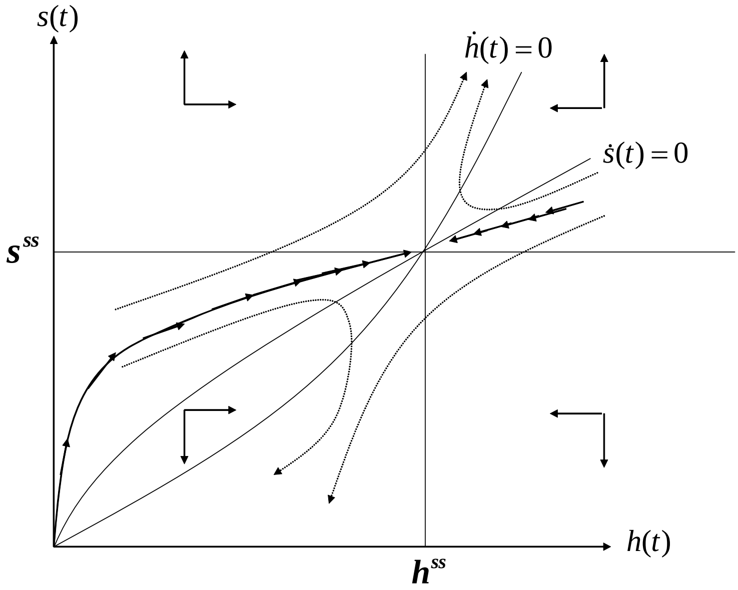


Fig. 2.— Phase dyagram for the case $\sigma > \overline{\sigma}$

to increase the public expenditure and personal transfers. From equation (18), it follows that the amount of skilled labor devoted to collect taxes (bureaucrats) rises throughout the transition until the skilled labor threshold, $\overline{h}$, is reached. This implies that: the effective tax rate, the amount of skilled labor used in the public education system (teachers) and the production of the public good and the personal transfers increase as well. However, once the bureaucratic sector reaches the critical size in which the effective tax rate equates to the statutory rate $\overline{\tau}$ at the threshold skilled labor $\overline{h}$, the government does not need to hire more bureaucrats and consequently, the per capita amount of skilled labor devoted to this sector remains stable. This allows the government to devote more resources to the public education system, with the consequent rise in the skilled labor in this sector up to its steady state level.

The amount of human capital, however, follows a different pattern. As we explained in section 6, in the first stage of development, the government is demanding skilled work for both programs bureaucray and public education system. If the requirements of skilled work for the government (in form of bureaucrats and teachers) are increasing in lower speed than the rate at which the total amount of human capital increases in the economy then, the remaining amount of human capital which is devoted to the production of private goods

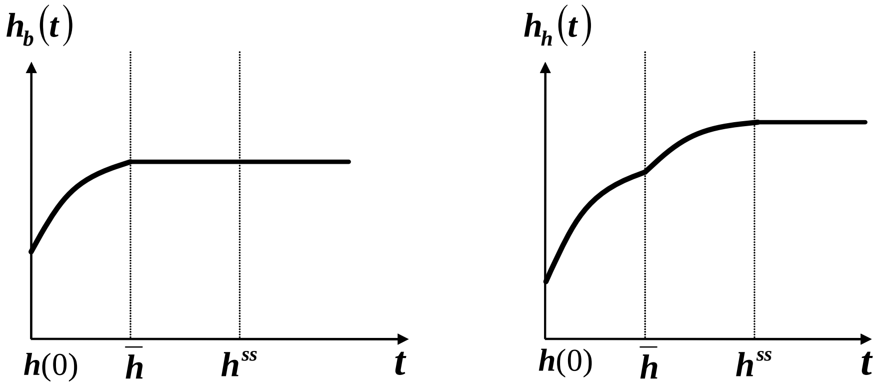


Fig. 3.—

increases at a decreasing rate. On the contrary, if the demands of the government of skilled labor is growing highly enough then, there would not be possible to continuing allocating human capital to the private sector and the amount of human capital in the production of goods will decrease. This is the case is which there exists a drain of human capital in the economy from the private sector to the public sector (see figure 1). When the size of the bureaucracy reachs its maximum level and the government is not hiring more bureaucrats, then the amount of skilled labor allocated in the private sector increases up to their steady state level. Figure 3 shows the evolution of the size of bureaucrats and teachers throughout the transition up to the steady state.

Notice that in the first stages of development, in the cases in which the government diverts human capital from the production of goods to the public sector, the production of goods declines. The reason is quite intuititive: during the transition to the steady state, not only it is growing the amount of skilled labor, also is increasing the number of students (proposition 2). This implies that the per capita amount of raw labor is declining in the equilibrium path. Thus, at the same time time that the goverment is draining human capital from the private sector to the public sector, the amount of raw labor used in the production of goods declines. In this case, clearly, the production of goods decreases. When the bureaucracy stabilizes and the government only hires teachers, in spite of the amount of unskilled labor continues decreasing, the per capita income increases since the amount

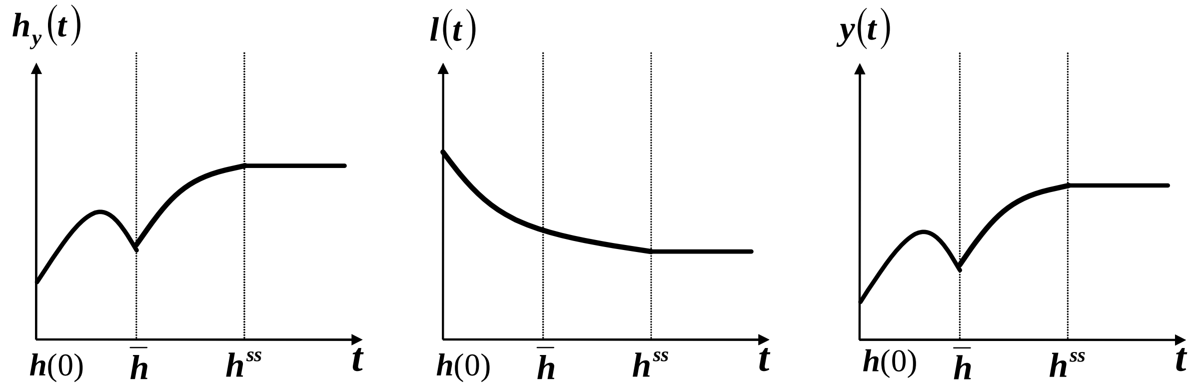


Fig. 4.—

of skilled labor in the production of goods increases. In this stage of development, the transition to the steady state would be characterized by increases in the skilled labor, the number of students and the per capita income. Figure 4 shows the effect of the evolution of the allocation of human capital in the production of goods when the government diverts skilled labor from the private to public sector. Morevoer, it shows the evolution of the unskilled labor and the per capita income throughout the transition to the steady state.

9. Implications of the model and empirical evidence

In this section we discuss the main implication of the proposed theory in the light of the literature and the empirical evidence available. These are mainly two: the tight link between public education and bureaucracy, and the relationship between skilled labor and the production of final goods.

9.1. Bureaucracy and public education

This model proposes a theory of reallocation of the skilled labor such that at the beginning of the development the government demands larger amount of human capital compared with later stages. The reason is that the government requires skilled labor to constitute the bureaucracy which allows it to collect taxes to finance a public good, public transfers and public education. Therefore, the government hires teachers to educate workers and, a share of these skilled workers will end up working for the government as teachers and as bureaucrats. The remaining amount of skilled workers would be devoted to the private sector producing goods.

Therefore, our theory suggests that the public education system is developed in societies at the same time as bureaucracy, since the government requires educated workers to carry out its activity. Because bureaucratic service requires literacy and some degree of mathematical knowledge, governments has incentive to provide education to population. This vision is shared by the mainstream literature of Political Science. Studies on the rise and evolution of the bureaucracy sector point out that governments demand competent administration, both to provide sufficient public goods to population to avoid removal from power (Bueno de Mesquita et al., 2003, Przeworski, Stokes and Manin, 1999) and/or to extract as much value from the population as possible (McGuire and Olson, 1996).

A groundbreaking paper by Hollyer (2011) perfectly illustrates this argument through the analysis of the change of the recruitment process in the bureaucratic sector in Western Europe. There have been two main methods of recruitment in bureaucracy: patronage and meritocracy. Patronage is the method of recruitment in which applicants offer some form of payment to the government in exchange for positions. Merit-based recruitment, on the other hand, makes positions available to those applicants that qualify based on some indicator of administrative performance. Medieval and early modern Europe was characterized by the patronage method of recruitment. Aristocracy and upper classes were favored for this system since they were the ones able to afford the financial cost of buying the positions. It was during the 19th and early 20th century when many countries faced a process of meritocratic reform. Hollyer (2011) argues that this revolutionary process was due to the widespread education of the population. The increase in the educational attainment among the middle and lower classes, typically excluded from the bureaucracy, led to a substantial rise in their administrative capabilities relative to those of upper classes. Thus, the opportunity cost of precluding the possibility of hiring this new type of skilled labor by the government under the patronage increased substantially, meaning that the adoption of merit reforms became more likely. He estimates a fixed-effects logit regression for a large set of countries and a long period, and he finds that 19th and early 20th century Western European governments were more likely to adopt the merit system as educational enrollment expanded. Moreover, he finds this relationship particularly pronounced in cases where the patronage mechanism was likely to be particularly exclusionary at initial stages.

Results from Hollyer (2011) are aligned with arguments of most traditional literature on the emergence of bureaucracy based on foundations from Max Weber. Max Weber (1978) contends that the process of rationalization and institutionalization of bureaucracy is a byproduct of economic development. He argues that economies can only grow and develop when the private sector can be certain of the impartial conduct of government functions. Also pointed out Gerth and Mills (1970) this kind of certainty is only present in bureaucratic systems that can be identified with the Weberian 'ideal type'.

9.2. Human capital and development

As we have commented previously, the weak relationship between skilled labor and economic performance evidenced by empirical macroeconomic literature is one of the puzzles that has received much attention by researchers in Development and Economic Growth. While there exists solid empirical evidence about the significant returns borne by education at the micro level (see Card, 1999, for a survey), at the macro level both cross-sectional studies (Kyriacou, 1991, Benhabib and Spiegel, 1994, Nonneman and Vanhoudt, 1996, Pritchett, 2001) and, more recently, panel data studies (Kumar, 2006, Bond, Hoeffler and Temple, 2001, Caselli, Esquivel and Lefort, 1996; Islam, 1995) report an insignificant or negative impact of skilled labor on economic growth. Moreover, Psacharopoulos (1994), Psacharopoulos and Patrinos (2004) and Strauss and Duncan (1995) evidence that the effects of education in developing countries is much higher than in developed countries and that it decreases with per capita income. This would imply that developing countries should grow at higher rate than developed countries but it is also well documented (see Easterly 1994, 2001), that such a high return on education does not always lead to a successful growth process. Therefore, these last observations suggest that the relationship between skilled labor and growth performance is even weaker in developing countries.

In section 2 we comment that, besides technical reasons and problems derived with the quality of the data, the economic literature has focused on looking for reasons which justify an "apparently" relatively low productivity of human capital in the economy (there are other crucial mechanisms affecting the development process as quality of institutions; skilled labor being allocated to unproductive activities or; education does not reflect skills). However, in our model, the reason why skilled capital does not have a significant effect on production at the beginning of development is because there is a delay in the use of it by the production sector. If the speed at which the amount of skilled labor grows is slower than the speed at which the government demands qualified workers to hire them as teachers and bureaucrats, then the amount of skilled labor available for the production sector declines. Therefore, the model predicts that per capita income might decrease during this early stage of development.

In this regard, Topel (1999) and Krueger and Lindahl (2001) provide evidence on the possible delay of human capital affecting economic growth. In analyzing the impact of increases in education on the growth rate, Krueger and Lindahl (2001) argue that including the change in physical capital as a control could bias the results downward if, because of complementarities between these factors, educational growth raises economic growth in part by fostering physical capital. So, contrary to many of the studies in the literature, they estimate a panel for a wide sample of countries leaving out the growth of physical capital and they find a significant association of education growth and GDP growth over long periods. More precisely they find that the change in schooling has little effect on GDP growth when the growth equation is estimated with high frequency changes (i.e., five years). However, increases in average years of schooling have a positive and statistically significant effect on economic growth over periods of ten or twenty years. Also, Topel (1999), estimating stylized growth models over varying length time intervals finds, similar results. Therefore, these studies are suggesting that considering short periods of time might not be appropriate to estimate the returns of education since they might appear later as our theory proposes.

10. Institutional changes

In this section we evaluate the effect of two important institutional reforms related with the performance of the government which have significant consequences on the development process: an improvement in the technology of bureaucracy and an increase in the size of the public education program.

10.1. The effect of an improvement in the technology of the bureaucracy

We analyze the effect of a technological improvement in the bureaucracy sector through an increase in the parameter Γ. In this context a technological improvement of the bureaucracy implies that for the same amount of bureaucrats, the tax collection increases, this is, the effective tax rate is closer to the legal tax rate.

An increase in Γ has a positive effect on household's disposable income. Given that bureaucrats are now more efficient, a smaller amount of skilled labor is now needed to

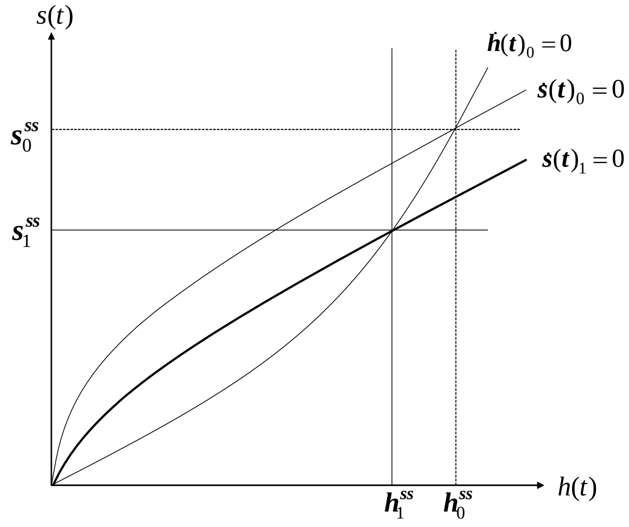


Fig. 5.—

collect taxes. Thus, for a given amount of tax collection, the government may devote more resources to the provision of income transfers to households and a flow of skilled labor from the bureaucracy to the production sector arises. The increase in the number of skilled workers in the private sector reduces the wage of skilled labor, discouraging the skilled labor accumulation and reducing the number of students. These effects graphically render in the movement of the locus $\dot{s} = 0$ to the right.

Proposition 5 *If there is a technological improvement in the bureaucracy sector, measured as an increase of* Γ*, then the steady state students and skilled labor levels decreses, the amount of human capital devoted to production and the production increases.*

Phase diagram in figure 5 shows the dynamic behavior of the economy due to an increase in Γ. From the initial the initial level of per capita skilled labor, there is a unique equilibrium path which converges to a steady state with a higher level of skilled labor and a larger number of students. Thus, a "structural change" throughout the transition occurs: the improvement in the technology of bureaucracy implies that a lower amount of skilled labor is required for bureaucracy, producing a reallocation of skilled labor from the bureaucratic sector to other sectors.

Figure 6 shows the evolution of the of skilled labor devoted to each sector of the economy when Γ increases at period t_0. Given the fact that bureaucrats are more efficient collecting

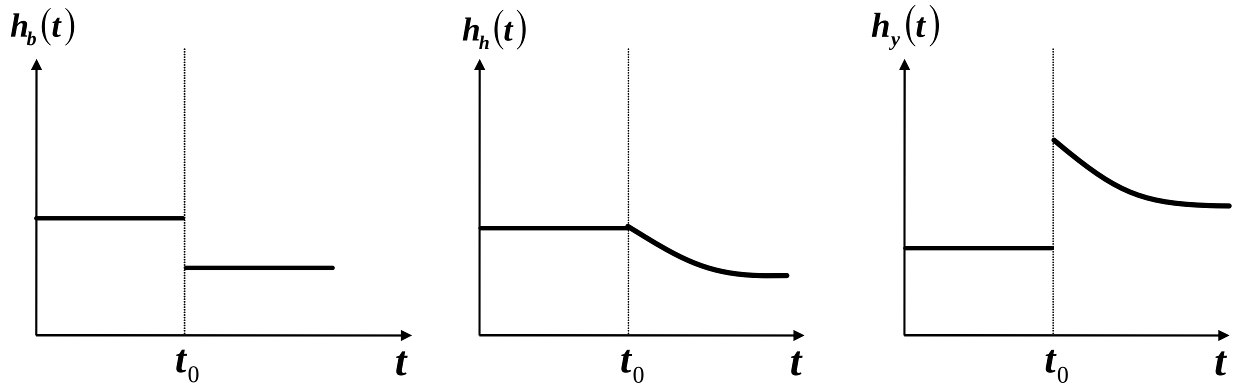


Fig. 6.—

taxes, the government can obtain the same amount of public revenues with a less number of bureaucrats. Thus, the government decides to hire less bureaucrats and to spend this amount of resources providing more income transfers to individuals (see equations 18 and 21); whereas skilled workers that leave the government are now reallocated in the production sector. The increase in the disposable income rises the household's levels of both consumption and number of students. Consequently, the amount of teachers and the number of skill workers which are in production sector increase as well. This allocation of human capital in the educational sector implies a positive effect of the return on the skilled labor, encouraging furthermore the skilled labor accumulation. Insofar skilled labor is being accumulated, the amount of it which is devoted to produce goods and to hire teachers grows.

10.2. The effect of an increase in the size of the public education system

We study the effect of an increase in the size of the public education program through a rise in the share of public revenue which is devoted to hire teachers, λ, for a given tax rate. An increase in λ produces a substitution effect: for the same number or bureaucrats and so, the same amount of tax collection, the government decides to spend a larger amount of public resources in hiring teachers at the expense of income transfers. Thus, a flow of skilled labor from the production sector to public sector arises, increasing the return on savings and so encouraging the skilled labor accumulation and the rise of the number of students. As a consequence, the wage of the skilled labor rises, increasing the benefits of investment in skilled labor furthermore and so, the incentive to accumulate more skilled labor and to rise the number of students. These effects graphically render in the movement of the locus $\dot{s} = 0$

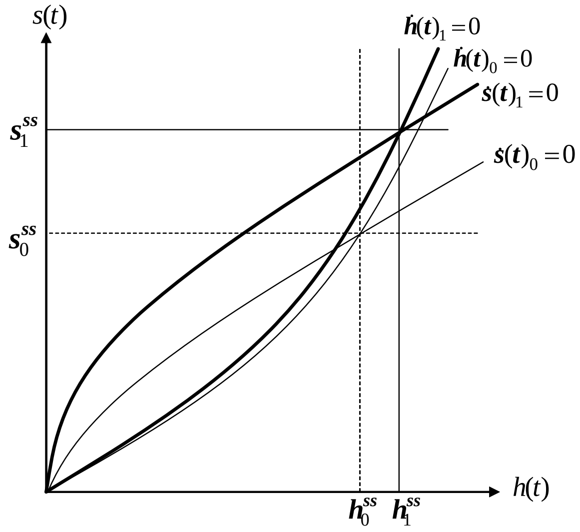
to the left.

Similarly, an increase in λ has also two opposite effects on the total amount of resources in the economy: first, in spite of households pay the same amount of taxes, the fact that the government hires more skilled labor at the expense of the skilled labor used in the production sector reduces the disposable amount of resources for households and so the consumption level and the number of students. Second, given that a larger fraction of the tax collection is devoted to increase the provision of teachers, the productivity of skilled labor in the production sector and in turn, the amount of disposable resources increase. The sign resulting from the combination of these two effects results ambiguous. Nevertheless, it is proved that the economy moves toward a new steady state with a higher number of students and a higher level of skilled labor.

Proposition 6 *If there is an increase in share of public revenues devoted to hire skilled labor, λ, then the steady state students and skilled labor levels increase as well.*

The dynamic behavior of the economy due to an increase in λ could be represented by a phase diagram similar to figure 5, where figure 7 (left) shows the case in which the effect of λ on the disposable amount of resources is positive (locus $\dot{h} = 0$ goes up) and figure 7 (right) shows the opposite (locus $\dot{h} = 0$ goes down). From the initial level of per capita skilled labor, there is a unique equilibrium path, which converges to the steady state with a higher level of skilled labor and a higher number of students. Thus, a "structural change" throughout the transition to the new steady state occurs: the increase in the share of public revenues devoted to hire teachers and to expand the public education system, implies a stable number of bureaucrats, an increase in the total amount of skilled labor and the number of students and, a higher level of consumption and per capita income.

Figure 8 shows the evolution of the of skilled labor devoted to each sector of the economy when λ increases at period t_0. For the given amount of tax collection, the government spends a higher amount of resources in teachers at the expense of a drop in the amount of income transfers provided, while the amount of skilled labor devoted to the bureaucratic sector remains stable. Since the public education system is more intensive in skilled labor than the production of the public good, the relative wage of skilled labor rises, involving a lower rate skilled-unskilled labor in the production sector. Thus, the amount of skilled labor devoted to the production of goods drops momentarily. However, given the fact that the higher amount of public education (teachers) increases the productivity of skilled labor and so, the return on it, a skilled labor accumulation process is generated. Insofar skilled labor is being accumulated, the amount of it which is devoted to produce goods and to hire teachers also increases throughout the transition to the new steady state.


s(t)
ḣ(t)₁ = 0
ḣ(t)₀ = 0
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h(t)
h₀ˢˢ
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s(t)
ḣ(t)₀ = 0
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ṡ(t)₁ = 0
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Fig. 7.—

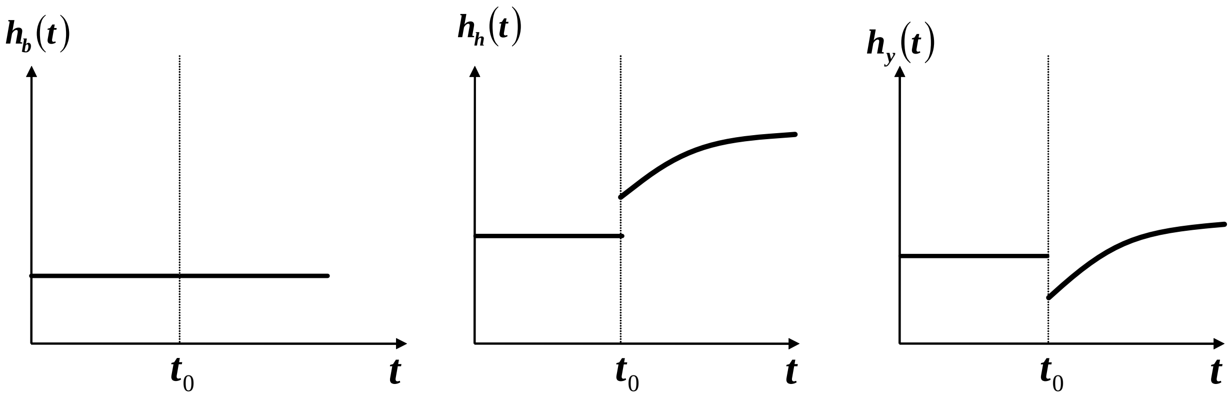
h_b(t)
t₀
t
h_h(t)
t₀
t
h_y(t)
t₀
t

Fig. 8.—

11. Conclusion

The role of the skilled labor in economic growth and development is one of the most analyzed topics in the economic literature. One of the main reason behind is the mixed and ambiguous evidence that the literature offers about the impact of skilled labor in the economic performance. While, at the micro level, empirical literature reports significant returns to increases in education consistent with the theory, the macro analysis finds not only a weak relationship between skilled labor and economic performance but also shows a negative impact of skilled labor. This combination of strong positive effect of skilled labor at the micro level with an ambiguous effect at macro level constitutes a puzzle which has attracted the interest of many researchers in Growth and Economic Development.

Simultaneously, the high return on skilled labor in developing countries and the low growth performance they have followed suggest that the dichotomy between the effects of the skilled labor at the micro and the macro level look likes more pronounced in developing countries and moreover, are partially responsible of the stable convergence in income levels across countries (see Easterly, 2001 and Parente and Prescott, 1993).

This paper offers a new explanation to understand why there exists a weak relationship between skilled labor and economic growth and why the low impact of skilled labor on economic growth is steeper in developing countries. This paper build a theory which explains how human capital is allocated in the economy during the development process and allow us to account for the main features related to the evolution of the human capital when economies are growing.

This paper suggests that the difficulties of governments of owning a educated bureaucracy to collect taxes and managing basic activities (provision of both public expenditure and income transfers) and; consequently, the fact that not all the skilled labor is devoted to public education or to produce goods is the reason that explains why, at least temporally, the skilled labor shows a low impact in the economy.

We build a model in which the public educational system is an important factor affecting the accumulation of skilled labor and where public revenues determine the amount of public expenditure on public education. The skilled labor is used to produce goods in the market, to create the government's bureaucracy and to constitute the government's provision of education in form of teachers. The level of tax collection depends positively on the size of the bureaucracy and a portion of public revenues is used to finance bureaucrats and to finance a public educational system (teachers) which is targeted to increase the skilled labor level. Thus, a feedback process arises: a higher level of skilled labor implies more efficient bureaucrats, who collect more taxes that are used to finance a public expenditure on education which in turn promotes skilled labor. Therefore, in the first stage, when the level of skilled labor is low, the scarcity of the skilled labor implies a high private returns of education, encouraging the skilled labor accumulation. Nevertheless, on the other hand, there is also a low level of tax collection which implies a low level of teachers in the economy. Given the fact that public education is a significant input in the production function of skilled labor, the low provision of education depress the return of skilled labor. This second mechanism on the return of skilled labor reduces partially the high private incentive to accumulate skilled labor, decelerating the skilled labor accumulation. Thus, when the level of skilled labor is low, income and tax collection are also low, implying low levels of public education and bureaucracy. Insofar skilled labor is growing throughout the transition to the steady state, the amount of collected taxes, the public expenditure on education and the size of bureaucracy also increase. There is a certain level of skilled labor in which the bureaucracy reach the point in which the tax collection is maximized, and the effective tax rate coincides with the statutory tax rate. After this level, the bureaucracy sector remains stationary and skilled labor is devoted only to the provision of the public education and to produce goods. The larger amount of skilled labor devoted to the educational sector will rise the productivity of the skilled labor and so, the accumulation of capital and the growth of the per capita income.

The paper also shows that the impact of the skilled labor in the economy depends crucially on the allocation of tax revenues to different uses, such as public educational system or the expenditure in public goods and income transfers. This allocation of the public resources not only has a temporal effect throughout the transition, but it may also affects in the long run. If the share of public revenues which is devoted to finance public education system increases then, a reallocation of public resources from expenditure in public transfers to the public education system arises, producing higher levels of consumption and skilled labor in the steady state. Thus, countries with the same initial skilled labor level, apparent similar access to technologies and identical characteristics could have different structural process and economic results depending on how the public sector allocates resources to different uses like public education or expenditure in public goods and income transfers.

Moreover, the model accounts for the well documented fact that the higher is the education of the bureaucracy, the better is the economic performance of the economy. However, it is worth to notice that the mechanism proposed in the paper does not excludes the possibility that some part of the bureaucrats may not be involved in unproductive or rent-seeking activities. On the contrary, it is proved that a more efficient bureaucracy produces a higher amount of public resources devoted to provide public skilled labor, an increase in the total amount of skilled labor and a higher level of consumption and per capita income. Therefore, according to the prediction of the model, the impact of skilled labor in the economy may be delayed by a low level of the bureaucracy efficiency.

Results and implications of the model are consistent with the availalbe empirical evidence. First, our theory states that in the first stage of development returns of skilled labor in the economy might be reduced since a large fraction of it is devoted to constitute the government, instead to stay in the production sector. Recent contributions of Topel (1999) and Krueger and Lindahl (2001) find that the sign and the strength of the relationship between skilled labor and economic performance depends on the time period considered. These papers seem suggest that considering short periods of time might not be appropriate to estimate the returns of education since they might appear later on, as our theory proposes. Thus, the aparent contradiction that many authors document between returns of education at the micro and macro level might dissapear if the considered span to measure is widened. Second, our theory shows that the bureaucracy system increases at the same time that the education system expands, and it demands a large amount of skilled labor is that early stage of development. When a optimal level of bureaucracy is reached, skilled labor flows largely to educational and the production sector. Hollyer (2011) shows strong evidence on this regard. He finds that bureaucracy disproporcionately expands in countries (measured thought the change of recruiment system from exclusive patronage to merit-based system) when education widespread among population. Thus, in a first stage of development, bureaucracy increases dramatically simulteously to the development of educational programs. This revolutionary process characterized the transition from the old governenmet to the beginning of modern ones in Western European countries. This results are aligned to the contributions of Weber (1971) which contends that the process of rationalization and institutionalization of bureaucracy is a byproduct of economic development.

Finally, we think this paper provides an interesting setting for analyzing the the role of the government and institutions in the skilled labor formation during the development process. The framework we propose, also allow us to study the feedback relationship between the main programs of the government thoughout different stages of development. We think our model might be used for further research in these areas. At theoretical level one may consider the possibility to endogenize the formation of institutions throughout the development process and to investigate at which extent institutions and skilled labor reinforce each other. Another possible extension of theoretical work is introducing corruption in the model and to analyze if there exists any feedback mechanism between the accumulation of skilled labor and the level of corruption and how it is evolving during the developing process. While at empirical level, there is need to look up to the point above which the returns of education are observable and have and may have positive and significant effect on growth and below which it has weak or negative effects on growth.

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13. Appendix

Proof of Proposition 3

It follows from eqs. (1), (9), (10) and (14) that at the steady state the following conditions should hold:

$$\left.\begin{array}{l} (h_h(h))^{\xi} s(t)^{1-\xi} = bh(t) \\ \left(\frac{h_h(h)}{s(t)}\right)^{\xi}\left[\frac{\alpha}{1-\alpha}\left(\frac{1-h-s}{h_y(h)}\right)(1-\tau(h_b(h))) - 1\right] = m+\rho \end{array}\right\} \Rightarrow$$

$$\left(\frac{h_h(h)}{bh(t)}\right)^{\frac{\xi}{1-\xi}}\left[\frac{\alpha(1-\tau(h_b(h)))}{1-\alpha}\left(\frac{1-h-\left(\frac{bh}{h_h(h)}\right)^{\frac{\xi}{1-\xi}} bh}{h_y(h)}\right) - 1\right] = m+\rho \tag{27}$$

Let’s define $\Psi(h)$ as the function that defines the steady state:

$$\Psi(h) = \left(\frac{h_h(h)}{bh(t)}\right)^{\frac{\xi}{1-\xi}}\left[\frac{\alpha(1-\tau(h_b(h)))}{1-\alpha}\left(\frac{1-h-\left(\frac{bh}{h_h(h)}\right)^{\frac{\xi}{1-\xi}} bh}{h_y(h)}\right) - 1\right] \tag{28}$$

Note that:

$$\lim_{h\to 0}\Psi(h) =$$

$$\lim_{h\to 0}\left(\frac{\lambda(\gamma^{\gamma}\Gamma)^{\frac{1}{1-\gamma}}}{b}\right)^{\frac{\xi}{1-\xi}} h^{\frac{\gamma\xi}{(1-\xi)(1-\gamma)}}\left[\frac{\alpha(1-\tau(h_b(h)))}{1-\alpha}\left(\frac{1-h-\left(\frac{b}{\lambda(\gamma^{\gamma}\Gamma)^{\frac{1}{1-\gamma}}}\right)^{\frac{\xi}{1-\xi}} bh^{\frac{1-\xi-\gamma}{(1-\xi)(1-\gamma)}}}{h-(\gamma+\lambda)(\gamma^{\gamma}\Gamma)^{\frac{1}{1-\gamma}} h^{\frac{1}{1-\gamma}}}\right) - 1\right] =$$

$$\lim_{h\to 0}\left(\frac{\lambda(\gamma^{\gamma}\Gamma)^{\frac{1}{1-\gamma}}}{b}\right)^{\frac{\xi}{1-\xi}} \frac{\alpha(1-\tau(h_b(h)))}{1-\alpha}\left(\frac{1-h-\left(\frac{b}{\lambda(\gamma^{\gamma}\Gamma)^{\frac{1}{1-\gamma}}}\right)^{\frac{\xi}{1-\xi}} bh^{\frac{1-\xi-\gamma}{(1-\xi)(1-\gamma)}}}{h^{\frac{1-\xi-\gamma}{(1-\xi)(1-\gamma)}}\left[1-(\gamma+\lambda)(\gamma^{\gamma}\Gamma)^{\frac{1}{1-\gamma}} h^{\frac{\gamma}{1-\gamma}}\right]}\right) = +\infty$$

where in the last equality we use the assumption that $\xi + \gamma < 1$. Let's define the function $\Phi(\Gamma)$ as follows:

$$\Phi(\Gamma) = \min_{h \in [0,\underline{h}]} \Psi(h;\Gamma) = \min_{h \in \left[0, \frac{\overline{\tau}^{\frac{1-\gamma}{\gamma}}}{\gamma \Gamma^{\frac{1}{\gamma}}}\right]} \Psi(h;\Gamma)$$

It is easy to check that the function $\Psi(h;\Gamma)$ it is increasing in Γ. Furthermore, $\lim_{\Gamma\to\infty} \underline{h} = 0$. Given that $\lim_{h\to 0} \Psi(h;\Gamma) = +\infty$ then, we can conclude that

$$\begin{aligned} \lim_{\Gamma\to\infty} \Phi(\Gamma) &\geq \lim_{h\to 0} \Psi(h;\widetilde{\Gamma}) = +\infty \Rightarrow \\ &\Rightarrow \lim_{\Gamma\to\infty} \Phi(\Gamma) = +\infty \end{aligned}$$

where $\widetilde{\Gamma} > 0$.

Let's define $\overline{\Gamma}$ as follows:

$$\overline{\Gamma} = \sup \{\Gamma \text{ s. th. } \Phi(\Gamma) = m + \rho\}$$

It follows from the definition of $\overline{\Gamma}$ and the fact that $\lim_{\Gamma\to\infty} \Phi(\Gamma) = +\infty$ that $\forall \Gamma > \overline{\Gamma}$, $\Phi(\Gamma) > m + \rho$. Thus, there is not steady state with $h \in [0, \underline{h}]$.

Note that if $h \geq \overline{h}$ and $\lambda\overline{\tau} \geq b$, then the function that defines the steady state (equation 28) is monotonically decreasing in h:

$$\left(\frac{h_h(h)}{bh(t)}\right)^{\frac{\xi}{1-\xi}} \left[\frac{\alpha(1-\overline{\tau})}{1-\alpha}\left(\frac{1-h-\left(\frac{bh}{h_h(h)}\right)^{\frac{\xi}{1-\xi}} bh}{h_y(h)}\right) - 1\right] =$$

$$\left(\frac{\overline{\tau}\lambda}{b}\right)^{\frac{\xi}{1-\xi}} \left[\frac{\alpha(1-\overline{\tau})}{1-\alpha}\left(\frac{1-h\left(1-b\left(\frac{b}{\overline{\tau}\lambda}\right)^{\frac{\xi}{1-\xi}}\right)}{(1-\overline{\tau}\lambda)h - \overline{h}_b}\right) - 1\right]$$

Furthermore, it follows from the definition of $\overline{\Gamma}$ that $\forall \Gamma > \overline{\Gamma}$:

$$\begin{aligned} \Psi(\overline{h};\Gamma) &> m + \rho \\ \Psi\left(\frac{1}{1-b\left(\frac{b}{\overline{\tau}\lambda}\right)^{\frac{\xi}{1-\xi}}};\Gamma\right) &= -\left(\frac{\overline{\tau}\lambda}{b}\right)^{\frac{\xi}{1-\xi}} \end{aligned}$$

Thus there is a unique $h^{ss} > \overline{h}$ such that $\Psi(h^{ss};\Gamma) > m + \rho$. $\Rightarrow\Leftarrow$ ■

Proof Proposition 4

When $h > \overline{h}$ the following equations hold in equilibrium:

$$
\begin{aligned}
w\left(h,s\right) &= (1-\alpha)A\left(\frac{\left(1-\overline{\tau}\lambda\right)h-\overline{h}_b}{1-h-s}\right)^{\alpha}; \quad w_h\left(h,s\right)=\alpha A\left(\frac{1-h-s}{\left(1-\overline{\tau}\lambda\right)h-\overline{h}_b}\right)^{1-\alpha}; \\
p_h\left(h,s\right) &= \frac{A(1-\alpha)s^{\xi}}{\left(\overline{\tau}\lambda\right)^{\xi}h^{\xi-\alpha}}\left(\frac{\left(1-\overline{\tau}\lambda\right)-\frac{\overline{h}_b}{h}}{1-h-s}\right)^{\alpha} = \frac{(1-\alpha)s^{\xi}}{\left(\overline{\tau}\lambda\right)^{\xi}h^{\xi}}\frac{y(h,s)}{1-h-s} \\
c\left(h,s\right) &= \left[\alpha\left(1-\frac{\varphi\overline{\tau}h}{\left(1-\overline{\tau}\lambda\right)h-\overline{h}_b}\right)\overline{\tau}+(1-\alpha)\right]A\left(\left(1-\overline{\tau}\lambda\right)h-\overline{h}_b\right)^{\alpha}(1-h-s)^{1-\alpha}
\end{aligned}
$$

Function $r(w_h\left(t\right), w\left(t\right))$ denotes the return of investment in skilled labor and it is defined from equation (14).

$$
r(w_h\left(t\right), w\left(t\right)) = \frac{w_h\left(t\right)\left(1-\tau(t)\right)-w\left(t\right)}{p_h(t)} + \frac{\dot{p}_h(t)}{p_h(t)}
$$

As above, when $h > \overline{h}$, we can rewrite it as:

$$
\begin{aligned}
r(h,s) &= \left(\frac{\overline{\tau}\lambda h}{s}\right)^{\xi}\frac{w_h\left(h,s\right)\left(1-\tau\right)-w\left(h,s\right)}{w\left(h,s\right)} = \\
&= \left(\frac{\overline{\tau}\lambda}{s}\right)^{\xi}\left[\frac{\frac{\alpha}{(1-\alpha)}\left(1-\overline{\tau}\right)\left(1-s\right)-\left(\frac{1}{1-\alpha}-\overline{\tau}\left(\lambda+\frac{\alpha}{(1-\alpha)}\right)\right)h+\overline{h}_b}{\left[\left(1-\overline{\tau}\lambda\right)-\frac{\overline{h}_b}{h}\right]h^{1-\xi}}\right]
\end{aligned}
$$

Then,

$$
\begin{aligned}
\frac{\frac{\partial c(h,s)}{\partial h}}{c\left(h,s\right)} &= \frac{\frac{\varphi\overline{\tau}\overline{h}_b}{\left(\left(1-\overline{\tau}\lambda\right)h-\overline{h}_b\right)^2}}{\left[\alpha\left(1-\frac{\varphi\overline{\tau}h}{\left(1-\overline{\tau}\lambda\right)h-\overline{h}_b}\right)\overline{\tau}+(1-\alpha)\right]} + \frac{w_h\left(h,s\right)\left(1-\lambda\overline{\tau}\right)-w\left(h,s\right)}{y\left(h,s\right)} > 0 \\
\frac{\frac{\partial p_h(h,s)}{\partial h}}{p_h\left(h,s\right)} &= -\frac{\xi}{h}+\frac{w_h\left(h,s\right)\left(1-\lambda\overline{\tau}\right)-w\left(h,s\right)}{y\left(h,s\right)}+\frac{1}{1-h-s} = \\
&= \left(1-\frac{\xi}{\alpha+\overline{h}_b\frac{w_h(h,s)}{y(h,s)}}\right)\frac{w_h\left(h,s\right)}{y\left(h,s\right)}+\frac{\alpha}{1-\alpha}\frac{w\left(h,s\right)}{y\left(h,s\right)} > 0
\end{aligned}
$$

where in the last inequality we used the assumption that $\xi > \alpha$. It follows from the above equations that:

$$\frac{\partial c(h,s)}{\partial h} > 0; \frac{\partial c(h,s)}{\partial s} < 0; \frac{\partial p_h(h,s)}{\partial s} > 0; \frac{\partial r(h,s)}{\partial h} < 0$$

The locus $\dot{s}(t) = 0$ is as follows:

$$G(h,s) = \left[\frac{\frac{\partial c(h,s)}{\partial h}}{c(h,s)} - \frac{1}{\sigma} \frac{\frac{\partial p_h(h,s)}{\partial h}}{p_h(h,s)} \right] F_h(h,s) - \frac{1}{\sigma}\left[r(h,s) - m - \rho \right] = 0$$

Note that:

$$\begin{aligned} \lim_{h \to h^{ss}} \frac{r(h,s^{ss}) - m - \rho}{F_h(h,s^{ss})} &= \lim_{h \to h^{ss}} \frac{r(h,s^{ss}) - m - \rho}{(\overline{\tau}\lambda)^{\xi} h^{\xi} (s^{ss})^{1-\xi} - bh} = \\ = \lim_{h \to h^{ss}} \frac{\frac{\partial r(h,s^{ss})}{\partial h}}{\frac{\xi(\overline{\tau}\lambda)^{\xi} h^{\xi}(s^{ss})^{1-\xi}}{h} - b} &= \frac{\frac{\partial r(h^{ss},s^{ss})}{\partial h}}{\frac{\xi b h^{ss}}{h^{ss}} - b} = -\frac{\frac{\partial r(h^{ss},s^{ss})}{\partial h}}{(1-\xi)b} > 0 \end{aligned}$$

Note that h^{ss} do not depend on σ. Furthermore:

$$\begin{aligned} \lim_{\sigma \to +\infty} \frac{\frac{\partial c(h^{ss},s^{ss})}{\partial h}}{c(h^{ss},s^{ss})} - \frac{1}{\sigma}\left(\frac{\frac{\partial p_h(h^{ss},s^{ss})}{\partial h}}{p_h(h^{ss},s^{ss})} - \frac{\frac{\partial r(h^{ss},s^{ss})}{\partial h}}{(1-\xi)b} \right) &= \frac{\frac{\partial c(h^{ss},s^{ss})}{\partial h}}{c(h^{ss},s^{ss})} > 0 \\ \lim_{\sigma \to 0} \frac{\frac{\partial c(h^{ss},s^{ss})}{\partial h}}{c(h^{ss},s^{ss})} - \frac{1}{\sigma}\left(\frac{\frac{\partial p_h(h^{ss},s^{ss})}{\partial h}}{p_h(h^{ss},s^{ss})} - \frac{\frac{\partial r(h^{ss},s^{ss})}{\partial h}}{(1-\xi)b} \right) &= -\infty \end{aligned}$$

Note that the above function is increasing in σ. Thus, let's define $\underline{\sigma}$ as follows:

$$\underline{\sigma} \Leftrightarrow \frac{\frac{\partial c(h^{ss},s^{ss})}{\partial h}}{c(h^{ss},s^{ss})} - \frac{1}{\underline{\sigma}}\left(\frac{\frac{\partial p_h(h^{ss},s^{ss})}{\partial h}}{p_h(h^{ss},s^{ss})} - \frac{\frac{\partial r(h^{ss},s^{ss})}{\partial h}}{(1-\xi)b} \right) = 0 \Leftrightarrow \underline{\sigma} \equiv \frac{\left(\frac{\frac{\partial p_h(h^{ss},s^{ss})}{\partial h}}{p_h(h^{ss},s^{ss})} - \frac{\frac{\partial r(h^{ss},s^{ss})}{\partial h}}{(1-\xi)b} \right)}{\frac{\frac{\partial c(h^{ss},s^{ss})}{\partial h}}{c(h^{ss},s^{ss})}}$$

If follows from the definition of $\underline{\sigma}$ that:

$$\text{if } \sigma < \underline{\sigma} \Rightarrow \frac{\frac{\partial c(h^{ss},s^{ss})}{\partial h}}{c(h^{ss},s^{ss})} - \frac{1}{\sigma}\left(\frac{\frac{\partial p_h(h^{ss},s^{ss})}{\partial h}}{p_h(h^{ss},s^{ss})} - \frac{\frac{\partial r(h^{ss},s^{ss})}{\partial h}}{(1-\xi)b} \right) > 0$$

When $h < h^{ss}$: $F_h(h,s^{ss}) > 0$, then the function $G(h,s)$, evaluated at s^{ss}, may be rewritten as follows:

$$G(h,s^{ss}) = \left[\left[\frac{\frac{\partial c(h,s)}{\partial h}}{c(h,s)} - \frac{1}{\sigma} \frac{\frac{\partial p_h(h,s)}{\partial h}}{p_h(h,s)} \right] - \frac{1}{\sigma} \frac{\left[r(h,s) - m - \rho \right]}{F_h(h,s)} \right] F_h(h,s)$$

It follows from the definition of $\underline{\sigma}$ that there is $\varepsilon < h^{ss} - \overline{h}$ such that $\forall h \in (h^{ss} - \varepsilon, h^{ss}) :$ $G(h, s^{ss}) > 0$. This implies that $\forall h \in (h^{ss} - \varepsilon, h^{ss}) : F_s(h, s^{ss}) > 0$ and $F_h(h^{ss}, s^{ss}) = 0$. Thus:

$$\frac{\partial F_s(h^{ss}, s^{ss})}{\partial h} \leq 0$$

Let's now define $s_{\dot{h}=0}(h) \Leftrightarrow F_h\left(h, s_{\dot{h}=0}(h)\right) = 0$; $s_{\dot{s}=0}(h) \Leftrightarrow F_h(h, s_{\dot{s}=0}(h)) = 0$. $F_s\left(h, s_{\dot{h}=0}(h)\right) = \frac{-\frac{1}{\sigma}\left[\frac{w_h\left(h, s_{\dot{h}}(h)\right) - w\left(h, s_{\dot{h}}(h)\right)}{p_h\left(h, s_{\dot{h}}(h)\right)} - m - \rho\right]}{\frac{\frac{\partial p_h\left(h, s_{\dot{h}}(h)\right)}{\partial s}}{p_h\left(h, s_{\dot{h}}(h)\right)} - \frac{\frac{\partial c\left(h, s_{\dot{h}}(h)\right)}{\partial s}}{c\left(h, s_{\dot{h}}(h)\right)}} < 0$. Thus, the locus $s_{\dot{s}=0}(h)$ is in between $s_{\dot{h}=0}(h)$ and the steady state level s^{ss}:

$$\forall h \in (h^{ss} - \varepsilon, h^{ss}) : \quad s_{\dot{h}=0}(h) < s_{\dot{s}=0}(h) < s^{ss}$$

Since $s_{\dot{s}=0}(h^{ss}) < s^{ss}$ it follows that the function $s_{\dot{s}=0}(h^{ss})$ should be increasing at h^{ss}. Furthermore, since $\forall h \in (h^{ss} - \varepsilon, h^{ss}) : \quad s_{\dot{h}=0}(h) < s_{\dot{s}=0}(h) < s^{ss} = s_{\dot{s}=0}(h^{ss}) = s_{\dot{h}=0}(h^{ss})$ it follows that:

$$\frac{\partial s_{\dot{h}=0}(h^{ss})}{\partial h} = -\frac{\frac{\partial F_h(h^{ss}, s^{ss})}{\partial h}}{\frac{\partial F_h(h^{ss}, s^{ss})}{\partial s}} \geq \frac{\partial s_{\dot{s}=0}(h^{ss})}{\partial h} = -\frac{\frac{\partial F_s(h^{ss}, s^{ss})}{\partial h}}{\frac{\partial F_s(h^{ss}, s^{ss})}{\partial s}} \geq 0$$

Since $\frac{\partial s_{\dot{h}=0}(h^{ss})}{\partial h} > 0$, it follows that one of the above inequalities is strict. These inequalities implies:

$$\left.\begin{array}{l} \frac{\partial s_{\dot{s}=0}(h^{ss})}{\partial h} = -\frac{\frac{\partial F_s(h^{ss}, s^{ss})}{\partial h}}{\frac{\partial F_s(h^{ss}, s^{ss})}{\partial s}} \geq 0 \\ \frac{\partial F_h(h^{ss}, s^{ss})}{\partial s} \leq 0 \end{array}\right\} \Rightarrow \frac{\partial F_s(h^{ss}, s^{ss})}{\partial h} \geq 0$$

The dynamic system in a surrounding of the steady state may be linearized as follows:

$$\begin{bmatrix} \dot{h}(t) \\ \dot{s}(t) \end{bmatrix} = \begin{bmatrix} \frac{\partial F_h(h^{ss}, s^{ss})}{\partial h} & \frac{\partial F_h(h^{ss}, s^{ss})}{\partial s} \\ \frac{\partial F_s(h^{ss}, s^{ss})}{\partial h} & \frac{\partial F_s(h^{ss}, s^{ss})}{\partial s} \end{bmatrix} \begin{bmatrix} h(t) - h^{ss} \\ s(t) - s^{ss} \end{bmatrix}$$

The eigenvalues are as follows:

$$\begin{vmatrix} \frac{\partial F_h(h^{ss}, s^{ss})}{\partial h} - \lambda & \frac{\partial F_h(h^{ss}, s^{ss})}{\partial s} \\ \frac{\partial F_s(h^{ss}, s^{ss})}{\partial h} & \frac{\partial F_s(h^{ss}, s^{ss})}{\partial s} - \lambda \end{vmatrix} = \lambda^2 - tr\,\lambda + Det$$

where:

$$\begin{aligned}
tr &= \frac{\partial F_h(h^{ss},s^{ss})}{\partial h} + \frac{\partial F_s(h^{ss},s^{ss})}{\partial s} > 0 \\
Det &= \underbrace{\frac{\partial F_h(h^{ss},s^{ss})}{\partial h}}_{-}\underbrace{\frac{\partial F_s(h^{ss},s^{ss})}{\partial s}}_{+} - \underbrace{\frac{\partial F_h(h^{ss},s^{ss})}{\partial s}}_{+}\underbrace{\frac{\partial F_s(h^{ss},s^{ss})}{\partial h}}_{-} = \\
Det &= \frac{\partial F_s(h^{ss},s^{ss})}{\partial s}\frac{\partial F_h(h^{ss},s^{ss})}{\partial s}\left[\frac{\frac{\partial F_h(h^{ss},s^{ss})}{\partial h}}{\frac{\partial F_h(h^{ss},s^{ss})}{\partial s}} - \frac{\frac{\partial F_s(h^{ss},s^{ss})}{\partial h}}{\frac{\partial F_s(h^{ss},s^{ss})}{\partial s}}\right] = \\
Det &= \underbrace{\frac{\partial F_s(h^{ss},s^{ss})}{\partial s}\frac{\partial F_h(h^{ss},s^{ss})}{\partial s}}_{+}\underbrace{\left[\frac{\partial s_{\dot{s}=0}(h^{ss})}{\partial h} - \frac{\partial s_{\dot{h}=0}(h^{ss})}{\partial h}\right]}_{-} < 0
\end{aligned}$$

Thus, one of the eigenvalues is positive and the another is negative. This means that the steady state is a saddle point. Furthermore, we have seem that $\forall h(t) \in (h^{ss}-\varepsilon, h^{ss})$ and $s(t) = s^{ss} : F_s(h, s^{ss}) > 0$, this implies that the converging path is such that $\forall h(t) \in (h^{ss}-\varepsilon, h^{ss}) : s(t) < s^{ss}$. Thus, throughout the converging path when $h(t) < h^{ss}$, the number of students $s(t)$ increases. ■

Proof proposition 5

It follows from equations (22), (23), (24) and (27) and proposition 3 that at the steady state:

$$\begin{aligned}
\left(\frac{\overline{\tau}\lambda}{b}\right)^{\frac{\xi}{1-\xi}}\left[\frac{\alpha(1-\overline{\tau})}{1-\alpha}\left(\frac{1-h-\left(\frac{b}{\overline{\tau}\lambda}\right)^{\frac{\xi}{1-\xi}}bh}{(1-\overline{\tau}\lambda)h-\overline{h}_b}\right)-1\right] &= m+\rho \Leftrightarrow \\
\left(\frac{1-h-\left(\frac{b}{\overline{\tau}\lambda}\right)^{\frac{\xi}{1-\xi}}bh}{(1-\overline{\tau}\lambda)h-\overline{h}_b}\right) &= \frac{\frac{m+\rho}{\left(\frac{\overline{\tau}\lambda}{b}\right)^{\frac{\xi}{1-\xi}}}+1}{\frac{\alpha(1-\overline{\tau})}{1-\alpha}} \Leftrightarrow \\
\ln\left(1-h-\left(\frac{b}{\overline{\tau}\lambda}\right)^{\frac{\xi}{1-\xi}}bh\right) - \ln\left((1-\overline{\tau}\lambda)h-\overline{h}_b\right) - \ln\left(\frac{m+\rho}{\left(\frac{\overline{\tau}\lambda}{b}\right)^{\frac{\xi}{1-\xi}}}+1\right) + \ln\left(\frac{\alpha(1-\overline{\tau})}{1-\alpha}\right) &= 0 \quad (29)
\end{aligned}$$

Apliying the Implicit Function theorem:

$$\frac{\partial h}{\partial \overline{h}_b} = -\frac{\frac{1}{(1-\overline{\tau}\lambda)h-\overline{h}_b}}{\frac{-h-\left(\frac{b}{\overline{\tau}\lambda}\right)^{\frac{\xi}{1-\xi}}bh}{1-h-\left(\frac{b}{\overline{\tau}\lambda}\right)^{\frac{\xi}{1-\xi}}bh} - \frac{(1-\overline{\tau}\lambda)}{(1-\overline{\tau}\lambda)h-\overline{h}_b}} > 0$$

Using equation (24), we may redefine equation (29) as follows:

$$h_y = (1-\overline{\tau}\lambda)\, h - \overline{h}_b \Leftrightarrow h = \tfrac{h_y+\overline{h}_b}{(1-\overline{\tau}\lambda)} \Rightarrow$$

$$\ln\left(1-\left[1+\left(\frac{b}{\overline{\tau}\lambda}\right)^{\frac{\xi}{1-\xi}} b\right]\frac{h_y+\overline{h}_b}{(1-\overline{\tau}\lambda)}\right) - \ln\left(h_y\right) - \ln\left(\frac{\frac{m+\rho}{\left(\frac{\overline{\tau}\lambda}{b}\right)^{\frac{\xi}{1-\xi}}}+1}{\frac{\alpha(1-\overline{\tau})}{1-\alpha}}\right) = 0$$

Apliying the Implicit Function theorem:

$$\frac{\partial h_y}{\partial \overline{h}_b} = -\frac{-\frac{\frac{1}{(1-\overline{\tau}\lambda)}}{1-\left[1+\left(\frac{b}{\overline{\tau}\lambda}\right)^{\frac{\xi}{1-\xi}} b\right]\frac{h_y+\overline{h}_b}{(1-\overline{\tau}\lambda)}}}{-\frac{\frac{1}{(1-\overline{\tau}\lambda)}}{1-\left[1+\left(\frac{b}{\overline{\tau}\lambda}\right)^{\frac{\xi}{1-\xi}} b\right]\frac{h_y+\overline{h}_b}{(1-\overline{\tau}\lambda)}} - \frac{1}{h_y}} < 0$$

Apliying the definition of $\overline{h}_b$, $\overline{h}_b = \frac{\overline{\tau}^{\frac{1}{\gamma}}}{\Gamma^{\frac{1}{\gamma}}}$ (see equation (18)), and the chain rule:

$$\frac{\partial h}{\partial \Gamma} = \underset{\oplus}{\frac{\partial h}{\partial \overline{h}_b}}\underset{\ominus}{\frac{\partial h_y}{\partial \Gamma}} < 0; \qquad \frac{\partial h_y}{\partial \Gamma} = \underset{\ominus}{\frac{\partial h_y}{\partial \overline{h}_b}}\underset{\ominus}{\frac{\partial h_y}{\partial \Gamma}} > 0$$

Finally, it follows from (1) that at the steady state:

$$s = \left(\frac{b}{\overline{\tau}\lambda}\right)^{\frac{\xi}{1-\xi}} bh \Rightarrow \frac{\partial s}{\partial \Gamma} = \left(\frac{b}{\overline{\tau}\lambda}\right)^{\frac{\xi}{1-\xi}} b\frac{\partial h}{\partial \Gamma} < 0 \Rightarrow \frac{\partial y}{\partial \Gamma} = \left[w_h - w\left(\frac{b}{\overline{\tau}\lambda}\right)^{\frac{\xi}{1-\xi}} b\right]\underset{\oplus}{\frac{\partial h}{\partial \Gamma}} > 0$$

Proof proposition:

Apliying the Implicit Function theorem to equation (29):

$$\frac{\partial h}{\partial \lambda} = -\frac{\frac{\frac{\xi}{1-\xi}\left(\frac{b}{\overline{\tau}\lambda}\right)^{\frac{\xi}{1-\xi}} bh\frac{1}{\lambda}}{1-h-\left(\frac{b}{\overline{\tau}\lambda}\right)^{\frac{\xi}{1-\xi}} bh} + \frac{\overline{\tau}h}{(1-\overline{\tau}\lambda)h-\overline{h}_b} + \frac{\frac{m+\rho}{\left(\frac{\overline{\tau}\lambda}{b}\right)^{\frac{\xi}{1-\xi}}}}{\frac{m+\rho}{\left(\frac{\overline{\tau}\lambda}{b}\right)^{\frac{\xi}{1-\xi}}}+1}\frac{1}{\lambda}}{\frac{-h-\left(\frac{b}{\overline{\tau}\lambda}\right)^{\frac{\xi}{1-\xi}} bh}{1-h-\left(\frac{b}{\overline{\tau}\lambda}\right)^{\frac{\xi}{1-\xi}} bh} - \frac{(1-\overline{\tau}\lambda)}{(1-\overline{\tau}\lambda)h-\overline{h}_b}} > 0$$

It follows from (1) that at the steady state:

$$s = \left(\frac{b}{\overline{\tau}\lambda}\right)^{\frac{\xi}{1-\xi}} bh \Rightarrow \frac{\partial s}{\partial \lambda} = \left(\frac{b}{\overline{\tau}\lambda}\right)^{\frac{\xi}{1-\xi}} b\frac{\partial h}{\partial \lambda} > 0$$