Like father, like son: Inheriting and bequeathing

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Abstract

This paper incorporates indirect reciprocal behavior in the context of bequeathing decisions into an otherwise standard OLG model. We provide conditions for the existence of a unique steady state with operative bequests. Contrary to standard OLG models, we show that taking into account such behavioral interactions allows one to rationalize both an increasing and U-shaped pattern of the inheritance to GDP ratio over time, consistent with recent empirical evidence. Moreover, the model predicts a non-linear (U-shaped) relationship between the size of an unfunded social security program and the long-run stock of per capita capital, which in turn provides a novel explanation of the inconclusive empirical findings on the relationship between social security, savings and long-run growth. Ricardian equivalence is shown to hold in a special case of the model.

Keywords: intergenerational transfers, indirect reciprocity, unfunded social security, overlapping generations, Ricardian equivalence
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1 Introduction

It is well known that bequests play an important role in determining aggregate savings and household wealth. Still, the underlying motivation for leaving bequests remains an unsolved problem. From a theoretical point of view, several bequest motives have been proposed to explain *altruistically* motivated transfers.[1] In these models, altruism is typically exogenously given, thus neglecting its source and evolutionary development.

However, it is by now well accepted that preferences, norms and cultural attitudes are partly formed as the result of heritable genetic traits (see e.g. Heckman (2006)), and partly transmitted through generations by a learning and socialization process or the imitation of role models (see e.g. Bisin and Verdier (2011) for a survey of the literature). With regard to altruism and bequeathing behavior, empirical evidence suggests that inheritances and intended bequests are indeed positively and significantly related even after controlling for a number of household characteristics, most importantly household net worth (Arrondel and Masson, 2001; Cox and Stark, 2005; Arrondel and Grange, 2014; Stark and Nicinska, 2015). These findings can be explained by the presence of indirect reciprocal behavior between three generations, a kind of interaction that has been found to be particularly important within family relations (Mauss, 1950/1990; Kohli and Künemund, 2003; Arrondel and Masson, 2006).[2] Having received an inheritance from one's parents may not only increase the probability of leaving a bequest by enhancing the disposable net wealth but also by shaping the attitude towards the pleasure of giving and towards leaving a bequest to one's own children. The aim of this paper is to incorporate such a behavior into an otherwise standard overlapping generations model where parents are concerned about the flow of bequest itself, as in Andreoni (1989).

Existing theoretical studies on altruistic preference formation focus either on direct re-

[1] See, e.g., Arrondel et al. (1997).

[2] The relationship between received inheritance and one's own bequest has likewise been studied by Arrondel et al. (1997) and Arrondel and Grange (2006) who find that the existence of transfers received from one's parents increases the probability to make the same kind of transfer to one's children. Arrondel and Masson (2006) note that indirect reciprocities provide a 'dynamic synthesis of altruism and exchange allowing [...] to introduce intermediate motivations for transfers which better fit the data'. Moreover, indirect evidence in favor of this kind of behavior is provided by Kopczuk and Lupton (2007) who find only little evidence to support either the altruistic or strategic bequest motive.

ciprocities concerning gifts (or donations) within a static framework (Stark and Falk, 1998), on indirect reciprocities in the context of private education financing (Kirchsteiger and Sebald, 2010) or on the relationship between altruism and long run growth when individuals may invest into the accumulation of altruism (Rapoport and Vidal, 2007). By contrast, this paper is the first to theoretically study indirect reciprocal behavior between three generations in the context of bequeathing decisions. Individuals do not directly reciprocate for the inheritance they have received from their own parents, but rather repay it by leaving an estate to their own children. In this way leaving a bequest does affect the immediate recipient, i.e. the next generation, as well as future generations.

Using this framework, we obtain three main results. First, we establish conditions for the existence of a unique steady state with either operative or inoperative bequests in the long run, depending on the relative weight individuals attach to their own old-age consumption and to their altruistic concern. Second, we show that the ratio of inheritance to GDP may follow an increasing (or U-shaped) pattern throughout the transition towards the steady state. This is consistent with recent well-know evidence on the long-run evolution of inheritance in France and other countries by Piketty (2011) and Piketty and Zucman (2014). While standard overlapping generations models typically predict a constant inheritance to GDP ratio at each point in time, the present paper demonstrates how the empirical findings can be rationalized within an OLG model when accounting for the existence of 'retrospective' intergenerational transfers (Arrondel and Masson, 2001). Third, we show that, in the present model, the relationship between the size of an unfunded social security system and the long-run stock of per capita capital is non-linear (U-shaped). Specifically, the overall effect is determined by the balance of two opposing effects: On the one hand the provision of social security provides additional old-age income and thus decreases young individuals motivation to save (the standard effect). On the other hand, however, a higher pension level increases the disposable income out of which to bequeath and thus the amount of bequest individuals receive from their parents, which in turn positively affects the child's attitude towards leaving a bequest to their own children. Consequently, individual savings and capital accumulation increase.[3] This novel and hitherto unexplored channel may contribute to explaining why existing empirical evidence on the relationship between the size of an un-

[3] Ricardian equivalence holds in the present model only if both effects exactly offset each other.

funded social security system and savings remains inconclusive, see e.g. Cigno and Rosati (1996), Ehrlich and Zhong (1998) or Zhang and Zhang (2004). In an extension, we illustrate that our main results are robust against adding accidental bequests to the model.

The remainder of this paper is organized as follows. Section 2 provides some further motivating evidence while section 3 presents the basic model and derives the main findings. Section 4 shortly concludes. All proofs and technical considerations are included in the appendix.

2 Motivating evidence

Economic theory has emphasized two competing models in order to explain intergenerational transfers within the family: Altruism, where transfers are made in order to even resources or well-being and smooth consumption between family members (Barro, 1974) and (self-interested) exchange, where parents use gifts or a promise of inheritance as payment for a child's services, support or 'attention' during their old age (Bernheim et al., 1985). However, extensive empirical work has not lead to conclusive evidence in favor of any of these two models, see e.g. the discussions in Zhang and Zhang (2001) or Arrondel and Masson (2006). Moreover, many observed phenomena, such as the intergenerational persistence of bequeathing patterns, are hard to explain with these models. For example, Arrondel and Masson (2006) demonstrate that individuals who have received a donation are twice as likely to give one themselves, and that the probability of helping one's children financially is 50% higher for individuals who were gift beneficiaries themselves. Furthermore, Arrondel and Grange (2014) find that individuals having inherited twice the average wealth of their generation, leave 35-60 % more to their own descendants than the average individual of the respective generation (after controlling for other household characteristics). Similarly, Stark and Nicinska (2015) use data from 14 European countries and find that the intention to bequeath is increased by both the experience of an inheritance as well as the expectation of inheriting. Finally, Wilhelm et al. (2008) show that generosity, which is closely linked to any form of altruistically motivated behavior, is positively correlated within families across generations.

The positive correlations of transfers between individuals from successive generations can be explained by indirect reciprocal behavior, the importance of which has been emphasized in sociological (Mauss, 1950/1990), biological (Nowak and Sigmund, 2005) and behavioral research (Seinen and Schram, 2006). In the context of bequeathing behavior this means that the cultural transmission of attitudes, values and norms across generations creates a pattern of behavior in which parents support their own children in a way similar to the way their parents treated them. As individuals cannot directly reciprocate for the received inheritance, they repay it by leaving an estate to their own children. Hence, the more inheritance parents have received themselves, the more they are willing to bequeath to their children.

Further evidence in favor of indirect reciprocal behavior in bequeathing decisions is provided by empirical findings related to wealth inequality and its persistence, see e.g. Bowles and Gintis (2002). Economic outcomes in terms of personal wealth accumulation are not only very similar across generations but also tend to be highly persistent over time. Recent estimates of intergenerational elasticities of wealth are in the order of 0.22, 0.37 or 0.32 to 0.43 for co-existing generations, i.e. before any transmission of wealth has taken place, see Arrondel (2013), Charles and Hurst (2003) and Mulligan (1997), respectively. However, while standard explanatory factors such as educational attainment, income or personal wealth may account for a large fraction of variation, still, 'almost 35 percent of the intergenerational wealth elasticity remains unexplained after income, propensity to own assets, education, gifts, and expected bequests are controlled for' (Charles and Hurst, 2003, p.1157). Similarly, Arrondel (2013) finds that those standard explanatory factors account for only 73% of total variation in intergenerational wealth elasticities. Therefore, taking into account the transmission of attitudes and traits towards bequeathing behavior may help to grasp a better understanding of individual behavior and also contributes to explaining persistence of wealth inequality across generations.

From a theoretical perspective, the most closely related studies are Stark and Falk (1998) and Kirchsteiger and Sebald (2010). In these papers, reciprocal behavior is formalized by introducing a so called 'empathy' function. Stark and Falk (1998), for example, model direct reciprocal behavior between two individuals by assuming that empathy is induced by gratitude which in turn depends on the size of the donation, the recipient's pre-transfer income, and the donor's pre-transfer income. They show that such a model is observationally indistinguishable from the standard altruism-motivated model. Similarly, Kirchsteiger and

Sebald (2010) incorporate indirect reciprocities with regard to educational investments into an overlapping generations model and examine their impact on human capital formation, well-being and education policies. In their model, the empathy function is assumed to depend on the level of the parents' stock of human capital. They demonstrate the existence of illiterateness traps which can however be overcome by compulsory schooling. In contrast to these studies, we formalize indirect reciprocal behavior in the context of bequeathing decisions and assume that the empathy function, i.e. the attitude towards leaving a bequest, depends on the size of the received inheritance, the generosity of the donor (the own parents) and recipient's need. Indeed, it is well known from experimental research in psychology that empathy induces altruism (the so called empathy-altruism hypothesis) and that a person in need is more likely to receive help, if the helper experienced more empathy (see e.g. Wilhelm and Bekkers (2010) and Batson (2011)).

In a broader sense, our research is related to an emerging field of economics that seeks to understand where preferences come from. In fact, the issue of preference formation has recently received much attention in the context of time-preference, consumption expenditure or risk and trust attitudes Becker and Mulligan (1997); Waldkirch et al. (2004); Dohmen et al. (2012). For example, Dohmen et al. (2012) provide evidence suggesting that parents who are more willing to take risks, or more willing to trust others, have children who are similarly risk tolerant and trusting. Consequently, attitudes and traits with regard to different economic key factors are determined to a substantial degree by an individual's parents. The current paper contributes to this strand of literature by exploring theoretically the role of indirect reciprocities with regard to bequeathing behavior.

Our work has also some connections to the literature on inherited tastes, which analyzes the transmission of preferences between generations. De la Croix (1996) and De la Croix and Michel (1999), for example, find that habit formation in consumption, which can (involuntarily) be passed from one generation to the next, may generate endogenous oscillations in a simple general equilibrium model. More recently, however, Gori and Michetti (2016) and Kaneko et al. (2016) not only show that these fluctuations disappear when endogenous fertility is considered, but also that it explains the observed decline of the fertility rate in developed economies. Finally, Galor and Oezak (2016) provide a well documented example that inherited tastes do have persistent effects. They establish that changes, which occurred in the Columbian Exchange, help to explain contemporary economic behavior in different areas such as health, education and savings. In a more closely related study, Alonso-Carrera et al. (2007) show that taste inheritance may affect bequest motives. They find that habits (based on individual's own consumption) reduce the strength of the bequest motive, while taste inheritance (based on parents' consumption) increases it. When habits are not considered and the bequest motive is operative, their qualitative results are similar to ours: an increase in the intensity of taste inheritance produces an increase in both savings and bequests in the steady state. However, they do not characterize the dynamics of savings and bequests throughout the transition towards the steady state and restrict their analysis to the case where the steady state equilibrium is unique and saddle-path.

3 The model

The basic framework is a two-period overlapping-generations model in the tradition of Diamond (1965) where the size of each generation is normalized to one. In the first period of life, each individual inelastically supplies one unit of labor and receives the wage w_t. She also receives a nonnegative bequest, b_t from her parents. Income is spent on consumption c_t and savings s_t:

$$w_t + b_t = c_t + s_t. \tag{1}$$

When old, each individual allocates the return to savings[4] $r_{t+1}s_t$ to second-period consumption d_{t+1} and to a nonnegative bequest to the offspring b_{t+1}:

$$d_{t+1} = r_{t+1}s_t - b_{t+1} \tag{2}$$

where r_{t+1} denotes the interest factor. Individual preferences are assumed to be of the Cobb-Douglas type and depend on first- and second-period consumption and on the amount of bequest devoted to the children. Consequently, the life-cycle utility function of an individual born in t is

$$U(c_t, d_{t+1}, b_{t+1}) = \ln c_t + \beta \ln d_{t+1} + \lambda_t \ln b_{t+1} \tag{3}$$

[4]For reasons of simplicity, we assume that capital depreciates completely in one period.

where $\beta > 0$ is a discount factor, and λ_t the degree of altruism which measures individual's attitude towards leaving bequest.[5] Consistent with the findings in Arrondel and Masson (2001), Cox and Stark (2005) and Stark and Nicinska (2015) (and the evidence cited in the introduction), we formalize the transmission of attitudes and traits with regard to leaving bequests, by assuming that parents' willingness to leave a bequest to their child depends on (i) the amount of inheritance they have received by their own parents b_t, (ii) the generosity of their own parents (as measured by a function of the inverse of the disposable income out of which to bequeath $(r_t s_{t-1})^{-\kappa}$) and (iii) their own (i.e. the parents', not the grandparents') need (as measured by a function of the inverse of the pre-transfer income $w_t^{-\phi}$). We thus define the attitude function as follows:

$$\lambda_t(b_t, w_t, r_t s_{t-1}) = \delta \frac{b_t}{w_t^{\phi}(r_t s_{t-1})^{\kappa}} \tag{4}$$

where $\delta > 0$ measures the strength of the transmission of attitudes from parents to children with respect to their bequeathing behavior and $\phi, \kappa \geq 0$. Clearly, if parents have received bequest from their own parents, they attach more importance to and feel more responsible for passing on bequest to their own child. Moreover, if a parent has not received any inheritance himself, he is not willing to leave a bequest to his child. Finally, all other things being equal, an increase in the parents' wage income, reduces their financial need and the share of inheritance income in total disposable income, which in turn lowers the strength of the bequeathing attitude. The same reasoning also applies to an increase in the grandparents' return to their savings as it lowers their perceived generosity.

The attitude function in equation (4) can be considered as a first attempt to formulate such a function in the growth literature. In defining the attitude function, we draw upon the standard formulation in the micro literature. Stark and Falk (1998), for example, use a very similar function with the unique difference that they model direct reciprocal behavior with respect to gifts. Thus, they relate the size of the donation to the endowments of both the recipient and the donor. Kirchsteiger and Sebald (2010) focus on indirect reciprocal behavior in the context of education decisions and, using a simpler formulation, consider the

[5] Introducing an explicit discount factor for the level of bequests which is left to the kin, e.g. by assuming $\beta\lambda_t \ln b_{t+1}$ in equation (3), would not change any of the qualitative results.

amount of parents' human capital as the key variable to account for individuals' attitudes on education. More recently, Stark and Nicinska (2015) formalize the attitude to bequeath by assuming that it is positively correlated with the experience of inheriting. However, in their formulation, individuals only care about the magnitude of the inheritance received by their parents independently of parents' effort, that is, parents' income and parents' received bequest.

We now turn to the individuals' utility maximization problem. Each individual maximizes the utility (3), subject to the constraints (1), (2) and to the nonnegativity of bequests ($b_{t+1} \geq 0$), by choosing c_t, s_t, d_{t+1}, and b_{t+1}. The first-order conditions of this maximization problem are

$$\frac{\partial U_t}{\partial s_t} = -\frac{1}{c_t} + \frac{\beta r_{t+1}}{d_{t+1}} = 0 \tag{5}$$

$$\frac{\partial U_t}{\partial b_{t+1}} = -\frac{\beta}{d_{t+1}} + \frac{\lambda_t(.)}{b_{t+1}} \leq 0 \quad (= 0 \text{ if } \quad b_{t+1} > 0) \tag{6}$$

In the following, we focus on interior solutions, which in turn requires $b_0 > 0$.[6] Solving the first order conditions yields optimal savings and the optimal amount of bequests:

$$s_t = \frac{\beta + \lambda_t(.)}{1 + \beta + \lambda_t(.)}(w_t + b_t) \tag{7}$$

$$b_{t+1} = \frac{\lambda_t(.)}{1 + \beta + \lambda_t(.)} r_{t+1}(w_t + b_t) \tag{8}$$

Both individual savings and optimal bequests depend positively on the disposable income, the amount of bequest transferred to the descendant and the degree of altruism $\lambda_t(.)$.

On the production side of the model, perfect competition between a large number of identical firms is assumed. A representative firm in period t produces a homogenous output good according to a Cobb–Douglas production function with capital K_t and homogeneous labor L_t as inputs:

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha}, \tag{9}$$

[6]Note that the model boils down to the standard overlapping generations model if individuals do not receive an inheritance and will thus not leave a bequest to their own child.

where $1 > \alpha > 0$ is the share parameter of capital. Profit maximization under perfect competition implies that, in equilibrium, production factors are paid their marginal products:

$$w_t = (1-\alpha)AK_t^{\alpha}L_t^{-\alpha} = (1-\alpha)Ak_t^{\alpha} \tag{10}$$

and

$$r_t = \alpha AK_t^{\alpha-1}L_t^{1-\alpha} = \alpha Ak_t^{\alpha-1} \tag{11}$$

where $k_t = K_t/L_t$ is the capital intensity.

Given initial values of the capital stock k_0 and the amount of bequest b_0, a perfect-foresight intertemporal equilibrium is characterized by a sequence of quantities and prices:

$$\{c_t,\ d_t,\ k_t,\ s_t,\ b_t;\ w_t,\ r_t\}_{t\geq 0}. \tag{12}$$

Individuals maximize utility, factor markets are competitive, and all markets clear. The market-clearing conditions for the labor, capital and good markets are[7]

$$L_t = 1, \tag{13}$$

$$K_{t+1} = s_t, \tag{14}$$

$$Y_t = c_t + s_t + d_t. \tag{15}$$

Hence, using equation (13), GDP per capita can be written as

$$y_t = Ak_t^{\alpha}. \tag{16}$$

Exogenous bequeathing attitude. As a benchmark, we first analyze the situation where the attitude towards giving is exogenously given, i.e. $\bar{\lambda} > 0$. In this case, the dynamics of

[7]Note that population size is normalized to one.

the model are fully characterized by the following first order difference equation:[8]

$$k_{t+1} = \frac{\bar{\lambda} + (1-\alpha)\beta}{1+\beta+\bar{\lambda}} A k_t^{\alpha} \tag{17}$$

Consequently, there exists a unique interior steady state which is globally stable:

$$\bar{k} = \left(\frac{\bar{\lambda} + (1-\alpha)\beta}{1+\beta+\bar{\lambda}} A \right)^{\frac{1}{1-\alpha}} \tag{18}$$

Moreover, the inheritance to GDP ratio, defined as $x_t \equiv b_t/y_t = b_t/(Ak_t^{\alpha})$, is constant over time, i.e.

$$x_{t+1} = \frac{b_{t+1}}{y_{t+1}} = \alpha \frac{\bar{\lambda}}{\beta + \bar{\lambda}}, \tag{19}$$

which is inconsistent with the increasing (or U-shaped) pattern found by Piketty (2011) and Piketty and Zucman (2014). A higher $\bar{\lambda}$ implies both larger savings and bequests in the steady state. This has also been found by Alonso-Carrera et al. (2007) when habits are not considered and the bequest motive is operative.

Endogenous bequeathing attitude. With an endogenous attitude function, however, the dynamics of the model are fully characterized by equations (7) and (8). Inserting factor prices ((10) and (11)), the capital market clearing condition (14) and the bequeathing attitude (4), these dynamics can be simplified as follows:

$$x_{t+1} = \alpha \frac{x_t k_t^{\alpha(1-\phi-\kappa)}}{\beta m + x_t k_t^{\alpha(1-\phi-\kappa)}} \tag{20}$$

$$k_{t+1} = \frac{\beta m + x_t k_t^{\alpha(1-\phi-\kappa)}}{(1+\beta)m + x_t k_t^{\alpha(1-\phi-\kappa)}} ((1-\alpha) + x_t) A k_t^{\alpha} \tag{21}$$

where $m = (1/\delta)(1-\alpha)^{\phi}\alpha^{\kappa}A^{\phi+\kappa-1}$. Further analysis of (20) and (21) then gives rise to the following proposition:

[8]To see this, insert (1) and (2) into (15), make use of (9), (10) and (11) to obtain $w_t + b_t = \frac{\bar{\lambda}+(1-\alpha)\beta}{\beta+\bar{\lambda}} A k_t^{\alpha}$ and recall (7). Note further that in this case the equilibrium definition does not require an initial value of bequests as the dynamics are completely determined by the evolution of the capital stock in each period.

Proposition 1 *The set of steady state equilibria is characterized as follows:*
• *There exists a trivial steady state with zero bequests:*

$$x^* = 0 \tag{22}$$
$$k^* = \left(\frac{\beta A(1-\alpha)}{1+\beta}\right)^{1-\alpha} > 0 \tag{23}$$

• *Regarding the existence of non-trivial steady states with operative bequests, we distinguish three possible cases:*

(i) Suppose that $\phi + \kappa = 1$ and that $\frac{\delta}{\beta} > \frac{(1-\alpha)^{\phi}}{\alpha^{1-\kappa}}$. Then, there exists a unique globally stable steady state as follows

$$k^* = \left(\frac{(1-\beta m)\alpha A}{m+\alpha}\right)^{\frac{1}{1-\alpha}} > 0 \tag{24}$$
$$x^* = \alpha - \beta m \geq 0 \tag{25}$$

(ii) Suppose that $\phi+\kappa < 1$. Then, there exists a critical strength of the transmission process, i.e.

$$\hat{\delta} \equiv \frac{\beta^2}{1+\beta} A^{\phi+\kappa} \frac{(1-\alpha)^{1+\phi}}{\alpha^{1-\kappa}} \left(\frac{\beta(1-\alpha)A}{1+\beta}\right)^{\frac{\alpha(\phi+\kappa)-1}{1-\alpha}} \tag{26}$$

such that the following holds: If $\delta > \hat{\delta}$, then there exists a unique interior steady state with operative bequests. By contrast, if $\delta < \hat{\delta}$, then there either exists a unique steady state with zero bequests (a poverty trap) or there are three steady states, one featuring inoperative bequests whereas in the remaining two bequests are positive.

(iii) Suppose that $\phi + \kappa > 1$ and that $k^* < \hat{k} \equiv \left(\frac{\alpha}{\beta m}\right)^{\frac{1}{\alpha(\phi+\kappa-1)}}$. Then, there exists a unique interior steady steady state with operative bequests.
Proof: *See appendix.*

The above result contrasts sharply with that of an economy with an exogenous altruistic degree $\bar{\lambda}$, where bequests will always be operative throughout the economy's transitional path and in the long run as long as parents' degree of altruism is positive, i.e., $\bar{\lambda} > 0$. With an endogenous bequeathing attitude, however, bequests are positive in the long run only if parents' valuation of their own old age consumption is not too high. This result is similar to the one derived from the dynastic altruism model, where bequests are positive only if the exogenously given degree of intergenerational altruism is sufficiently strong (see e.g. Weil (1987)).

Note that the parameters ϕ and κ, which measure the relative importance of the individual's own need and the parents' generosity in the attitude function, respectively, play a similar role in determining the steady state level of the capital stock. Whereas both types of income (i,e., capital and labor income) depend positively on the per capita capital level, factor income shares remains stable over time and thus also the relative importance of both arguments in the attitude function.[9] Moreover, if $\phi + \kappa = 1$, one can show that higher values of ϕ and κ imply a larger steady state level of capital, i.e., $sign(\partial k^*/\partial\phi) = -sign(\ln(1-\alpha)) > 0$ and $sign(\partial k^*/\partial\kappa) = -sign(\ln(\alpha)) > 0$, as individuals increase savings in order to leave a larger amount of bequest.

The existence of multiple steady states may be used to explain why economies that differ with regard to their initial level of per capita income and/or their initial endowment of bequests end up at different steady states where bequests are either operative and the level of capital accumulation is therefore relatively large or bequests are inoperative and individuals are primarily focussed on satisfying their own consumption needs which in turn implies low levels of savings and capital accumulation. In the latter situation, successive generations may be trapped in poverty. While it is well-known that economic models with externalities in preferences or technology may exhibit multiple steady states (see, e.g., Azariadis and Drazen (1990) or Michel et al. (2006)), the channel highlighted in the present paper, namely changes in the attitude towards leaving bequests and indirect reciprocal behavior, has not been discussed before.

Finally, we shortly discuss the issue of stability. Proposition 1 shows that there is a unique globally stable steady state when $\phi + \kappa = 1$. Simulation results for the case $\phi + \kappa \neq 1$ suggest that if there exists a unique steady state (with either operative or inoperative bequests), this steady state is globally stable. By contrast, if there are two interior steady states (which requires $\phi + \kappa < 1$), one of them and the steady state with inoperative bequests are locally stable. This case is illustrated in Figure 1 for the parameters $A = 1$, $\alpha = 0.5$, $\beta = 0.5$,

[9] More sophisticated production functions would produce different results. An augmenting labor intensity technology that increases the labor income share, for instance, would increase the relevance of the parameter κ, that is, the importance of the own need.

$\phi = 0.1$ and $\kappa = 0.1$. More precisely, Figure 1 shows simulation results for varying initial levels of per capita income and bequest with $\delta = 3.64$. For these parameter choices the critical value of the strength of the transmission process equals $\hat{\delta} \approx 3.65$, so that $\delta < \hat{\delta}$ and the model exhibits two interior steady states. Depending on the initial level of per capita income and the endowment of bequest, the system converges either towards the steady state $k_* = 0.104$ with operative bequests, $b_* = 0.066$, or to the steady state $k_* = 0.028$ with inoperative bequests, $b_* = 0$ (the unstable steady state is given by $k_* = 0.029$ and $b_* = 0.0007$).

[Insert Figure 1 here]

The second main result is obtained by further examining the dynamics of the ratio of inheritance over GDP (equation (20)):

Proposition 2 *The ratio of inheritance over GDP may be increasing (U-shaped) over time.*
Proof: *See appendix.*

To illustrate these results, consider the following numerical examples. One period in the model is assumed to last half a generation, i.e., 30 years. Moreover, the capital income share α and the discount factor β are fixed at 0.3 and 0.4, respectively. While the first number is a standard value in the literature, the second one implies a plausible annual discount factor of 0.97 (see e.g. Gonzalez-Eiras and Niepelt (2008)). After normalizing $A = 1$, we can choose initial values of bequest b_0 and per capita capital k_0 in order to match actual observed inheritance to GDP ratios. Specifically, we consider two cases: First, we match the inheritance to GDP ratio in France in 1920 which was equal to 9.8% (see Piketty (2011, Fig.1)) by setting $b_0 = 0.022$ and $k_0 = 0.007$, and calculate the predicted ratios for the years 1950, 1980 and 2010 (recall that one model period is equal to 30 years). The parameters of the attitude function, are set to $\delta = 3.33$, $\phi = 0.01$ and $\kappa = 0.2$, respectively. These choices imply an initial number of $\lambda_t(.) = 0.13$. Second, we match the inheritance to GDP ratio in Germany in 1960 which was equal to 2% (see Piketty and Zucman (2014, Fig.4.5)) by setting $b_0 = 0.02$ and $k_0 = 1$, and calculate the models' predictions for the years 1990 and 2020 (using $\phi = 0.5$ and $\kappa = 0.3$ which in turn implies $\lambda_t(.) = 0.12$).

[Insert Figures 2 and 3 around here.]

Figures 2 and 3 compare the evolution of actual inheritance to GDP ratios in France and Germany with the models' predictions. Clearly, the numerical values show similar patters as the ones observed in the data (even though the drop of the inheritance to GDP ratio during the period of the first and second world war is underestimated). In sum, the model with an endogenous bequeathing attitude may account for both an increasing and U-shaped pattern of the inheritance to GDP ratio consistent with recent empirical evidence on the long-run evolution of inheritance (Piketty, 2011; Piketty and Zucman, 2014).[10]

What is the intuition behind these findings? As can be inferred from equation (19), a higher level of the bequeathing attitude $\lambda_t(.)$ increases the inheritance to GDP ratio as individuals save more in order to leave a larger amount of bequest to their children. The bequeathing attitude in turn is affected by the state variables of the economy as follows: A larger amount of received inheritance b_t unambiguously increases the attitude towards bequeathing oneself, whereas a higher capital stock k_t lowers $\lambda_t(.)$ because it reduces both the parents' need and the grandparents' generosity. Hence, the overall change in the inheritance to GDP ratio x_t depends on the relative change in k_t and b_t (according to equation (A.11)).[11] Consequently, when per capita capital is low, interest rates are high and wealth coming from the past (i.e. received by one's grandparents) matters more than wealth accumulated out of current income (see equation (8)). By contrast, when capital accumulates at a higher rate, interest rates decrease and the ratio of inheritance to GDP may fall as wealth accumulated out of current income becomes relatively more important. Such a non-linear relationship between the amount of inheritance and per capita income is not present in standard models of intergenerational transfers and, thus, provides a first distinct and testable prediction of the current model.

[10] We have checked that the dynamics of both patterns depicted in figures 2 and 3 converge towards a unique stable steady state, respectively.

[11] Formally, this can be seen by inserting factor prices ((10) and (11)) and the capital market clearing condition (14) into the bequeathing attitude (4), which yields $\lambda_t(k_t, b_t)$. Equation (A.10) can then be rewritten as

$$dx_{t+1} = \frac{\partial x_{t+1}}{\partial \lambda_t} \left(\frac{\partial \lambda_t}{\partial k_t} dk_t + \frac{\partial \lambda_t}{\partial b_t} db_t \right) \tag{27}$$

with $\partial x_{t+1}/\partial \lambda_t > 0$, $\partial \lambda_t/\partial k_t = -\delta \alpha b_t (\kappa + \phi) k_t^{-1} (\alpha A k_t^{\alpha})^{-\kappa} ((1-\alpha) A k_t^{\alpha})^{-\phi} < 0$ and $\partial \lambda_t/\partial b_t = \delta (\alpha A k_t^{\alpha})^{-\kappa} ((1-\alpha) A k_t^{\alpha})^{-\phi} > 0$.

3.1 Unfunded social security

The aim of this section is to illustrate that the present model may not only account for recent empirical findings regarding the dynamics of the inheritance to GDP ratio but in addition yields novel predictions with respect to the impact of public policies. This is particularly important in the context of pension policies as recent empirical evidence on the relationship between the size of a social security system and the level of per capita capital in the economy turns out to be mixed. For example, using cross sectional data, Ehrlich and Zhong (1998) find that the size of unfunded social security has a negative impact on savings and growth whereas Zhang and Zhang (2004) find no effect for the same relationship. By contrast, Cigno and Rosati (1996) find a positive effect of public pension on savings using time series data for Germany, Italy, UK and USA, respectively. Finally, and more recently, Bruce and Turnovsky (2013) study a calibrated OLG model and show that steady-state economies with pay-as-you-go social security programs grow more slowly than those without.

Standard OLG models fail to offer a clear explanation of this evidence. While it is well known that pay-as-you-go social security systems lower the steady state level of per capita capital (which is welfare enhancing in the case of dynamically inefficient economies), we show that in the present model a more complex relationship between the size of the social security system and the per capita level of capital may emerge if bequests are operative. To do so, we introduce a pay-as-you-go social security system and study its effect on long-run capital accumulation. For reasons of simplicity, we focus on the case with a unique stable steady state throughout this section, i.e. $\phi + \kappa = 1$, and therefore set $\kappa = 1 - \phi$.

Introducing a pay-as-you-go pension scheme alters individuals' budget constraints as follows:

$$(1-\tau)w_t + b_t = c_t + s_t \tag{28}$$

$$d_{t+1} = r_{t+1}s_t - b_{t+1} + \theta_{t+1} \tag{29}$$

where τ is the contribution rate and θ_{t+1} the pension benefit in old age. A balanced government's budget in each period t requires

$$\theta_{t+1} = \tau w_{t+1}. \tag{30}$$

Moreover, the pension scheme affects the attitude function as it alters the parents' disposable income (it increases their need) and also the grandparents' disposable income out of which to bequeath (and therefore the grandparents' generosity). The modified attitude function can thus be written as follows:[12]

$$\lambda_t(b_t, (1-\tau)w_t, r_t s_{t-1} + \theta_t) = \delta \frac{b_t}{((1-\tau)w_t)^{\phi}(r_t s_{t-1} + \theta_t)^{1-\phi}} \tag{31}$$

Solving the model in the same way as before gives the following system of first order difference equations:

$$x_{t+1} = (\alpha + \tau(1-\alpha))\frac{\lambda_t(.)}{\beta + \lambda_t(.)} \tag{32}$$

$$k_{t+1} = \frac{\alpha(\beta + \lambda_t(.))}{\alpha(1+\beta+\lambda_t(.)) + (1-\alpha)\tau}((1-\tau)(1-\alpha) + x_t)Ak_t^{\alpha} \tag{33}$$

With an exogenously given bequeathing attitude, i.e. $\bar{\lambda} > 0$, it is straight forward to show that an unfunded pension program unambiguously lowers capital accumulation.[13] In this case, the sole effect of providing social security is to reduce private savings and capital accumulation as it negatively affects the youngs' motivation to save. By contrast, with an endogenous bequeathing attitude, the effect critically depends on whether or not bequests are operative in the long run: Specifically, inserting the attitude function (31) into (32) and (33) and rearranging both equations yields:

$$x_{t+1} = (\alpha + \tau(1-\alpha))\frac{x_t}{\beta\tilde{m} + x_t} \tag{35}$$

$$k_{t+1} = \frac{\alpha(\beta\tilde{m} + x_t)}{\alpha[(1+\beta)\tilde{m} + x_t] + (1-\alpha)\tau\tilde{m}}((1-\tau)(1-\alpha) + x_t)Ak_t^{\alpha} \tag{36}$$

[12]Note that if the attitude function is assumed to be independent of the contribution rate, it is straight forward to show that the Ricardian equivalence holds as long as bequests are operative.

[13]To see this rearrange equations (32) and (33) in steady state to obtain

$$\bar{k} = \left[\frac{\alpha(\bar{\lambda} + \beta(1-\alpha)(1-\tau))}{\alpha(1+\beta+\bar{\lambda}) + (1-\alpha)\tau}A\right]^{\frac{1}{1-\alpha}} \tag{34}$$

and note that $\partial\bar{k}/\partial\tau < 0$.

where $\tilde{m} = (1/\delta)[(1-\tau)(1-\alpha)]^{\phi}[\alpha+\tau(1-\alpha)]^{1-\phi}$. Note that this system of two equations is qualitatively very similar to the one describing the baseline economy (equations (20) and (21) with $\phi+\kappa=1$). Hence, it is straight forward to show that the results of proposition 1 regarding existence, uniqueness and stability of steady states and the pattern of the inheritance to GDP ratio proved in proposition 2, also hold in the extended model with social security. The unique stable steady state is given by:

$$x^* = \max(0, \alpha+\tau(1-\alpha)-\beta\tilde{m}) \tag{37}$$

$$k^* = \left(\frac{\alpha(\beta+\frac{x^*}{\tilde{m}})}{\alpha(1+\beta)+(1-\alpha)\tau+\frac{x^*}{\tilde{m}}}((1-\tau)(1-\alpha)+x^*)A\right)^{\frac{1}{1-\alpha}} \tag{38}$$

Further analysis of (37) and (38) gives rise to the following proposition:

Proposition 3 *Suppose that bequests are operative in the long run, i.e.* $\frac{\delta}{\beta} > (\frac{(1-\alpha)(1-\tau)}{\alpha+\tau(1-\alpha)})^{\phi}$. *Then, there exists a non-linear (U-shaped) relationship between the size of an unfunded pension program and the long run stock of per capita capital, i.e.*[14]

$$\frac{\partial k^*}{\partial \tau} \gtreqless 0 \quad \Leftrightarrow \quad \tau \gtreqless \frac{1-\alpha-\phi}{1-\alpha} \equiv \hat{\tau} \tag{39}$$

By contrast, if bequests are inoperative, i.e. $\frac{\delta}{\beta} < (\frac{(1-\alpha)(1-\tau)}{\alpha+\tau(1-\alpha)})^{\phi}$, *an increase in the public pension program unambiguously reduces capital accumulation, i.e.* $\partial k^*/\partial\tau < 0$.
Proof: *See appendix.*

While it is well-know that social security lowers capital accumulation in the standard OLG model without bequests, the interesting and novel result of proposition 3 is the non-linear relationship between the contribution rate and the long-run stock of per capita capital when bequests are operative. In particular, Ricardian equivalence holds in a special case when $\phi = (1-\tau)(1-\alpha)$ or, equivalently, if $\tau = (1-\alpha-\phi)/(1-\alpha)$. There are several offsetting effects.[15] First, the provision of social security lowers individual savings and thus capital accumulation

[14] For example, suppose $\alpha = 0.3$ and $\phi = 0.5$. Then, $\hat{\tau} = 0.286$. Furthermore, if $\phi > 1-\alpha$, an unfunded social security program is always beneficial for capital accumulation as $\hat{\tau} < 0$ and thus $\partial k^*/\partial\tau > 0$.

[15] To see this formally, note that $\partial(x^*/\tilde{m})/\partial\tau > 0$ and $\partial x^*/\partial\tau > 0$ if $\tau > (1-\alpha-\phi)/(1-\alpha)$ and compare equation (38) with equation (33).

(the standard effect which is also present when bequests are inoperative). Second, the contribution rate affects the long-run capital stock through changes in the attitude function $\lambda_t(.)$. Specifically, a larger τ implies a higher level of the attitude function in steady state, which is in turn beneficial for savings and capital accumulation.[16] Finally, a higher pension level tends to increase the disposable income out of which to bequeath and thus increases the amount of bequest individuals receive from their parents and therefore their own disposable income and savings. The balance of these countervailing effects determines the relationship between the size of the pension program and the long-run stock of per capita capital.[17]

Similar to the results from the previous section, the findings of proposition 3 stand in sharp contrast to that of an economy with an exogenous bequeathing attitude. It is consistent, however, with non-monotonic patterns found in models of endogenous growth (see, e.g., Zhang and Zhang (1998) or Lambrecht et al. (2005)). Furthermore, it allows to rationalize the different and inconclusive empirical findings on how pay-as-you-go pensions affect savings and long-run growth. Therefore, our results also complement more recent contributions by Fanti (2014, 2015) in the context of the neoclassical overlapping generations model and the one with endogenous growth, respectively. However, the reason behind a U-shaped relationship between the size of the social security and the long-run stock of capital in these papers is the relative size of the capital share. More precisely, they show that a reduction in the mandatory retirement age may favour economic growth and even pension payments when the capital share is sufficiently high.

Summarizing, the model presented in this subsection provides a second novel and empirically testable prediction which is distinct from previous standard OLG models with intergenerational transfers, in which altruism is typically assumed to be exogenously given. Hence, our overall findings emphasize the importance of taking into account these social interactions to gain an improved understanding of individual behavior with potentially interesting and

[16] The overall positive effect of the contribution rate τ on the attitude function $\lambda_t(.)$ can be further decomposed into a direct positive effect resulting from an increase in the parents' need (due to a lower net income), a direct negative effect resulting from a decline in the grandparents' perceived generosity (due to a higher disposable income out of which to bequeath) and an ambiguous effect resulting from general equilibrium changes in the share of inheritance factor prices.

[17] Extensive numerical simulations show that the U-shaped relationship is also present if $\phi + \kappa \neq 1$. Assuming, for example, $A = 1$, $\alpha = 0.3$, $\beta = 0.4$, $\delta = 3.33$, $\phi = 0.5$ and $\kappa = 0.3$, yields $\hat{\tau} = 0.11$. Similarly, with $\phi = \kappa = 1.2$, we obtain $\hat{\tau} = 0.3$.

novel implications regarding the impact of public policies on economic outcomes.

3.2 Accidental bequest

Many empirical studies document that a significant part of bequest are not intended but, instead, accidental (see, for instance, Hurd (1997)). The aim of this section is to analyze whether our previous findings are robust to adding accidental bequest to our model. To do this, we use a similar setting as Pecchenino and Pollard (1997) and Fanti et al. (2014).

Specifically, we assume the length of the second period to be uncertain so that each agent is either dead or alive at the beginning of the second period with probability $1-\pi$ and π, respectively. For simplicity, we assume that $\beta = 1$ as in Pecchenino and Pollard (1997). Consequently, the life-cycle utility function of an individual born in t is given by

$$U(c_t, d_{t+1}, b_{t+1}) = \ln c_t + \pi \ln d_{t+1} + \lambda_t \ln b_{t+1} \tag{40}$$

Since agents do not know the time of death, accidental bequests can occur. If an agent dies at the onset of old age, her wealth is bequeathed uniformly to all of her children. Thus, unintentional bequests of parents born at period t and received children born in period $t+1$ are defined as

$$\gamma_{t+1} = (1-\pi) r_{t+1} s_t \tag{41}$$

Similarly, the first-period budget constraint is now modified to include the accidental bequest received when young

$$w_t + b_t + \gamma_t = c_t + s_t \tag{42}$$

whereas the budget constraint in the second period remains unchanged

$$d_{t+1} = r_{t+1} s_t - b_{t+1}. \tag{43}$$

As in the previous section, each individual maximizes the utility function (40), subject to the constraints (42), (43), (41) and to the nonnegativity of bequests ($b_{t+1} \geq 0$), by choosing c_t, s_t, d_{t+1}, and b_{t+1}. Solving the first order conditions yields optimal amounts of both savings and bequests:

$$s_t = \frac{\pi + \lambda_t(.)}{1 + \pi + \lambda_t(.)}(w_t + b_t + \gamma_t) \tag{44}$$

$$b_{t+1} = \frac{\lambda_t(.)}{1 + \pi + \lambda_t(.)} r_{t+1}(w_t + b_t + \gamma_t) \tag{45}$$

Note that these solutions are very similar to the ones defined by equations (7) and (8) in the model without accidental bequest. Since the probability of being alive at the second period π now captures the intensity on how agents discount the future, the unique difference between this model and the model without accidental bequest is that resources in the first period are increased by the amount of unintentional bequests.

Inserting factor prices ((10) and (11)), the capital market clearing condition (14), the bequeathing attitude (4) and the definition of accidental bequests (41), the dynamics can be written as follows:

$$x_{t+1} = \alpha \frac{x_t k_t^{\alpha(1-\phi-\kappa)}}{\pi m + x_t k_t^{\alpha(1-\phi-\kappa)}} \tag{46}$$

$$k_{t+1} = \frac{\pi m + x_t k_t^{\alpha(1-\phi-\kappa)}}{(1+\pi)m + x_t k_t^{\alpha(1-\phi-\kappa)}}((1-\pi\alpha) + x_t)Ak_t^{\alpha} \tag{47}$$

with m being defined as before.

Note that those dynamics of the economy are technically identical to the system describing the baseline economy (equations (20) and (21)). Consequently, we can conclude that our findings described in proposition 1, as well as the pattern of the inheritance to GDP ratio proved in proposition 2, do also hold in the model with accidental bequest. Similarly, proposition 3 in the version of the model with a PAYG system should also carry over to the extended version. A more careful analysis of these cases would be an interesting issue for future research.

4 Conclusions

This is the first paper to study indirect reciprocal behavior between three generations in the context of bequeathing decisions within a dynamic model of overlapping generations. Contrary to standard OLG models which typically consider only two generations (parents and children), we show that taking into account such behavioral interactions allows one to rationalize both an increasing and U-shaped relationship between the level of inheritance and per capita income consistent with recent empirical evidence (Piketty, 2011; Piketty and Zucman, 2014). Furthermore, the model predicts a non-linear (U-shaped) relationship between the size of an unfunded social security program and the long-run stock of per capita capital. This finding may help to explain why existing empirical evidence on such a relationship remains inconclusive in general. The present model could be extended to study the positive and normative implications of inheritance taxation (see e.g. Piketty and Saez (2013) for a recent overview) or other related fiscal policies such as public debt. Moreover, introducing a more explicit intergenerational transmission mechanism of bequeathing behaviour would be valuable.

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Appendix

Proof of proposition 1:
In a first step, we study the properties of the functions

$$F(k) = k - \frac{\beta(1-\alpha)}{(1+\beta)} A k^{\alpha} \tag{A.1}$$

and

$$G(k) = k^{1-\alpha(\phi+\kappa)}/A - k^{\alpha(1-\phi-\kappa)} + k^{1-\alpha}\frac{m}{\alpha A} + \beta m \tag{A.2}$$

which result from setting $x_t = 0$ and assuming $k_{t+1} = k_t = k$ in equation (21), and from inserting the steady state version of equation (20) into (21) and rearranging terms, respectively. The domain of both functions F and G is restricted to the interval $[0, \infty($. It is then straight forward to show that F has a local minimum at $\tilde{k} = \left(\frac{\alpha\beta(1-\alpha)A}{(1+\beta)}\right)^{\frac{1}{1-\alpha}}$. Moreover, we have $F(0) = 0$, $F(k_*) = 0$ with $k_* = \left(\frac{\beta(1-\alpha)A}{(1+\beta)}\right)^{\frac{1}{1-\alpha}}$, which proves the first part of proposition 1, and $\lim_{k\to\infty} F(k) = \infty$.
Now consider the function G. Possible roots of G are implicitly given by the solutions of the following equation

$$RHS(k) \equiv k^{\alpha(1-\phi-\kappa)} - k^{1-\alpha(\phi+\kappa)}/A = k^{1-\alpha}\frac{m}{\alpha A} + \beta m \equiv LHS(k) \tag{A.3}$$

with $LHS(0) = \beta m > 0$ and $\lim_{k\to\infty} LHS(k) = \infty$. Moreover, $LHS(k)$ is strictly concave.

Further analysis of $RHS(k)$ shows that

$$\lim_{k\to\infty} RHS(k) = \begin{cases} -\infty, & \phi+\kappa \leq 1/\alpha \\ 0, & \phi+\kappa > 1/\alpha \end{cases} \tag{A.4}$$

and

$$\lim_{k\to 0} RHS(k) = \begin{cases} 0, & \phi+\kappa < 1 \\ 1, & \phi+\kappa = 1 \\ \infty, & \phi+\kappa > 1 \end{cases} \tag{A.5}$$

Moreover, solving $RHS'(k) = 0$ yields $k_{**} = \left(\frac{\alpha(1-\phi-\kappa)A}{1-\alpha(\phi+\kappa)}\right)^{\frac{1}{1-\alpha}}$. Straight forward calculations show that k_{**} is a local maximum if $\phi + \kappa \leq 1$, a local minimum if $\phi + \kappa > 1/\alpha$ whereas $RHS'(k)$ is monotonically decreasing if $1 < \phi + \kappa < 1/\alpha$. Consequently, depending on the parameters of the model, G has either no roots, one root or two roots.

Given the properties of the functions $LHS(k)$ and $RHS(k)$, it follows that $\lim_{k\to\infty} G(k) = \infty$ and

$$\lim_{k\to 0} G(k) = \begin{cases} > 0, & \phi+\kappa < 1 \\ -\infty, & \phi+\kappa > 1 \end{cases} \tag{A.6}$$

In a second step, we consider three different cases.

First, suppose that $\phi + \kappa = 1$. In this case, the dynamics of the inheritance to GDP ratio are independent of the capital stock. Consequently, as the left hand side of (20) is concave in x_t, it is straight forward to see that there exist a globally stable steady state $x^* = \max(0, \alpha - \beta\bar{m})$. Note further that the condition for $x^* \gtrless 0$, i.e. $\alpha - \beta\bar{m} \gtrless 0$, is equivalent to the slope of the function x_{t+1} (i.e. equation (20) with $\phi + \kappa = 1$) at $x_t = 0$ being larger or smaller than one. Recalling the definition of $\bar{m}$ and solving $x^* \gtrless 0$ gives $\frac{\delta}{\beta} \gtrless \frac{(1-\alpha)^\phi}{\alpha^{1-\kappa}}$. Moreover, from (21) it is clear that once x_t has reached its steady state value, the dynamics of k_t converge monotonically towards k^* (as given in (24)).

Second, suppose that $\phi + \kappa < 1$ and consider the piecewise function

$$H(k) = \begin{cases} F(k), & k < \hat{k} \equiv \left(\frac{\beta m}{\alpha}\right)^{\frac{1}{\alpha(1-\phi-\kappa)}} \\ G(k), & k > \hat{k} \end{cases} \tag{A.7}$$

on the domain $[0, \infty($, with $\hat{k}$ being determined by solving the steady state version of equation (20):

$$x^* \gtrless 0 \quad \Leftrightarrow \quad (\hat{k})^{\alpha(1-\phi-\kappa)} \gtrless \frac{\beta m}{\alpha}. \tag{A.8}$$

$H(k)$ accounts for the fact that bequests can not be negative and its roots correspond to the steady states of our model. Straight forward calculations show that $F(\hat{k}) = G(\hat{k})$. Given the properties of the functions F and G, it follows that there exists a unique steady state with operative bequests whenever $F(\hat{k}) < 0$. By contrast, there may be either one steady state with inoperative bequests or three steady states if $F(\hat{k}) > 0$. The latter inequality can be rewritten as follows:

$$F(\hat{k}) \gtrless 0 \quad \Leftrightarrow \quad \delta \lesseqgtr \frac{\beta^2}{1+\beta} A^{\phi+\kappa} \frac{(1-\alpha)^{1+\phi}}{\alpha^{1-\kappa}} \left(\frac{\beta(1-\alpha)A}{1+\beta}\right)^{\frac{\alpha(\phi+\kappa)-1}{1-\alpha}} \equiv \hat{\delta}$$

By example we show that both cases, one steady state or three steady states, are indeed feasible. Consider the following parameterizations: $\alpha = 0.5$, $\beta = 0.5$ and $\delta = 3.4$, $\delta = 3.64$ or $\delta = 3.66$, respectively. In the first case ($\delta = 3.4$), $H(k)$ has a unique root at $k^* = 0.028$ whereas for $\delta = 3.64$ $H(k)$ exhibits three roots at $k^* = 0.028$, $k^* = 0.029$ and $k^* = 0.104$. Finally, for $\delta = 3.66$ there is exactly one root at $k^* = 0.109$.

Third, suppose that $\phi + \kappa > 1$ and consider the piecewise function $\bar{H}(k)$

$$\bar{H}(k) = \begin{cases} G(k), & k < \hat{k} \equiv \left(\frac{\alpha}{\beta m}\right)^{\frac{1}{\alpha(\phi+\kappa-1)}} \\ F(k), & k > \hat{k} \end{cases} \tag{A.9}$$

From the properties of the functions F and G it is clear that there is exactly one interior steady state with operative bequests if $k^* < \hat{k}$ whereas there is a unique steady state with inoperative bequests if $k^* > \hat{k}$.

Proof of proposition 2:

Insert the definition of $x_t = b_t/(Ak_t^\alpha)$ into equation (20) and express x_{t+1} as a function $g(k_t, b_t)$. Totally differentiating this latter function gives

$$dx_{t+1} = \frac{\partial g}{\partial k_t} dk_t + \frac{\partial g}{\partial b_t} db_t \tag{A.10}$$

with

$$\frac{\partial g}{\partial k_t} = -\frac{(\phi+\kappa)\beta m \alpha^2 A k_t^{\alpha(\phi+\kappa)-1} b_t}{(\beta m A k_t^{\alpha(\phi+\kappa)} + b_t)^2} \quad \text{and} \quad \frac{\partial g}{\partial b_t} = \frac{\beta m \alpha A k_t^{\alpha(\phi+\kappa)}}{(\beta m A k_t^{\alpha(\phi+\kappa)} + b_t)^2}$$

Solving $dx_{t+1} \gtreqless 0$ gives

$$\frac{db_t/b_t}{dk_t/k_t} \gtreqless \alpha(\phi+\kappa). \tag{A.11}$$

Proof of proposition 3:

The threshold for operative (inoperative) bequests is obtained by rearranging $x^* \geq 0$. Similarly to the condition in proposition 1, the condition $\alpha + \tau(1-\alpha) - \beta\tilde{m} \gtreqless 0$, is equivalent to the slope of the function x_{t+1} (i.e. equation (32)) at $x_t = 0$ being larger or smaller than one. Consider first the case with operative bequests. Insert (37) into (38) and simplify terms to obtain:

$$k^* = \left[\frac{\alpha}{\alpha\delta + \gamma}(\delta - \beta\gamma)A\right]^{\frac{1}{1-\alpha}}$$

where $\gamma = (\alpha + \tau(1-\alpha))^{1-\phi}((1-\tau)(1-\alpha))^\phi$. It is then straight forward to see that $\partial k^*/\partial\gamma < 0$ and

$$\frac{\partial \gamma}{\partial \tau} \gtreqless 0 \quad \Leftrightarrow \quad \tau \lesseqgtr \frac{1-\alpha-\phi}{1-\alpha}.$$

Now consider the case when bequests are zero. In this case, (38) simplifies to

$$k^* = \left[\frac{\alpha\beta(1-\alpha)(1-\tau)}{\alpha(1+\beta)+(1-\alpha)\tau}A\right]^{\frac{1}{1-\alpha}}$$

and straight forward calculations show that $\partial k^*/\partial \tau < 0$.

Figures

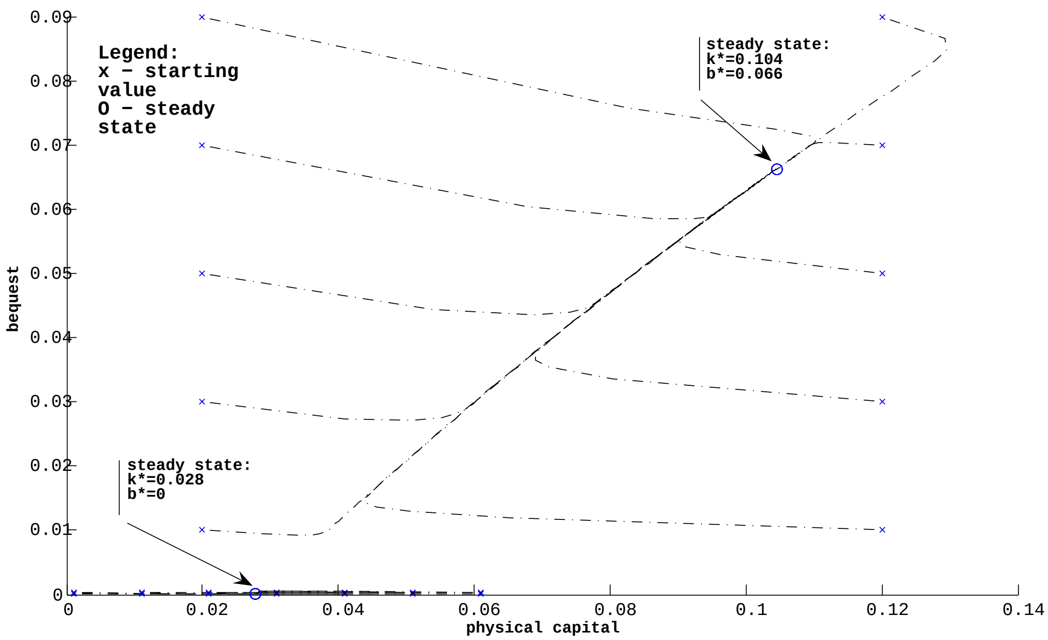

Figure 1: Existence and stability of multiple steady states

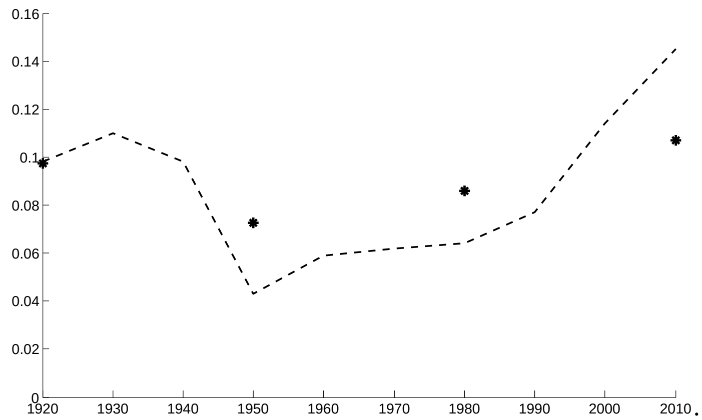

Figure 2: Inheritance to GDP ratio, France 1920-2010: Predicted (*) and actual (Piketty (2011, Fig. 1), --).

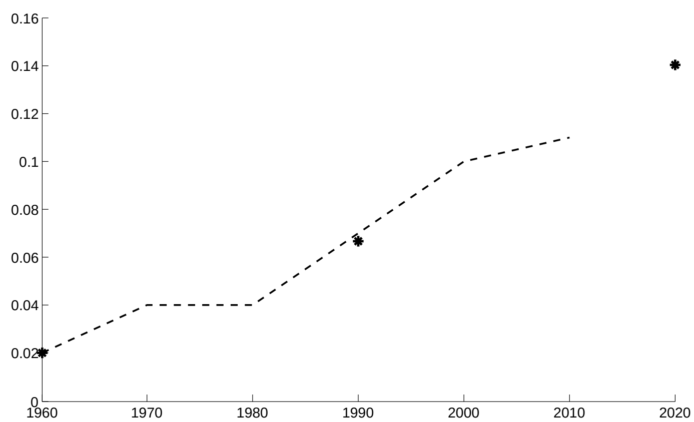

Figure 3: Inheritance to GDP ratio, Germany 1960-2020: Predicted (*) and actual (Piketty and Zucman (2014, Fig. 4.5), --).